LITHOSPHERIC STRENGTH PROFILES

In order to study the mechanical response of the lithosphere to various types of forces, one has to take into account its **rheology**, which literally means how it flows. This depends on the thermal structure, the fluid content, the thickness of compositional layers and various boundary conditions. Three main rheological modes determine the flow behaviour of Earth materials: elasticity, plastic yielding and viscous creep. In assessing flow processes in the lithosphere, each of these three modes will be considered. But an important factor is the time during which the load is applied.

- At time scale of seismic waves (up to hundreds seconds) the sub-crustal mantle behaves elastically down to deep within the asthenosphere.
- Over a few to thousands of years (e.g. ice cap), the mantle flows like a viscous fluid.
- On long geological times (more than 1 million year), the upper crust and the upper mantle behave also as thin elastic plates that overlie an inviscid (i.e. with no viscosity) substratum.

Both elasticity and viscosity are therefore ingredients of the mechanical behaviour of the lithosphere. The long-term mechanical attributes of the lithosphere are expressed in terms of **lithospheric strength**. This strength is estimated by integrating yield stress with depth. The current state of knowledge of rock rheology is sufficient to provide broad general outlines of mechanical behaviour, but also has substantial limitations. Two very thorny problems involve the scaling of rock properties with long time periods and for very large length scales.

ELEMENTS OF RHEOLOGY

Definitions

Rheology describes the response of materials to an imposed stress system. This response varies considerably according to the physical conditions of deformation. The **flow** behaviour of rocks to applied loads is empirically derived from laboratory experiments and can be compared to theoretical **constitutive equations**.

Gross geological differentiation

By experimentally subjecting rocks to forces and stresses under controlled conditions, one can observe, and describe mathematically, the nature of the deformation and the specific relationships between stress and strain, more precisely between the rate of application of stress and the rate of deformation. Different rock types respond differently to the forces that act upon them. The response of each rock type depends on the conditions under which the force is applied. As a general observation:

- Under low confining pressures and temperatures like those at shallow depths in the crust, and on short time scales, the sample returns to its original dimensions when the load is removed (the material behaviour is **elastic**) or has deformed by fracturing (the material is **brittle**).
- Under high confining pressures and temperatures like those at greater depths in the crust, the sample deformed, slowly and steadily without fracturing. It behaved as a pliable or mouldable material, that is, it deformed ductilely. The deformed sample does not return to its original dimensions when the load is removed. At least part of the strain is permanent. The sample behaved as a **plastic** material.

In summary, experiments show over all that cold rocks in the upper part of the crust are brittle and hot rocks in the lower part of the crust are ductile. This approximation will permit to outline some of the mechanisms of deformation that are thought to exist in the lithosphere.

Constitutive equations

Rheological equations are called constitutive equations because they describe time-dependent stress/strain behaviours resulting from the internal constitution of the material, such as thermal energy, pore fluid pressure, grain size, etc., and other external parameters such as temperature, etc.

In other words, constitutive equations involve material (**intrinsic**) properties such as composition, mass, density, and other external (**extrinsic**) conditions such as pressure, temperature, chemistry of the environment etc. For each constitutive equation, a mechanical analogue will be considered. However, there is no single equation that describes the material behaviours over the wide range of physical conditions. One must consider a number of ideal classes of response (**rheologies** such as elastic, viscous, plastic, etc.) which some materials display to various degrees of approximation under various physical conditions. For these reasons it is important to distinguish between **materials** and **response**.

Material

Mechanical properties

Homogeneity

Materials may be mechanically **homogeneous** or **inhomogeneous**. A strictly homogeneous material is one in which all pieces are identical. In other words, material composition and properties are independent of position.

Isotropy

Homogeneous materials may be mechanically **isotropic** or **anisotropic**. An isotropic material is one in which the mechanical properties are equal in all directions. In other words, material properties are independent of the direction in which they are measured. Examples of homogeneous and isotropic materials are sandstones and granites.
Layered and foliated rocks are statistically homogeneous, anisotropic materials if the scale of the layering or fabric is small relative to the scale of deformation.

Parameters

Material parameters are quantities that define some characteristics. Parameters such as elasticity, rigidity, compressibility, viscosity, fluidity are actually no material constants. They are scalars in isotropic materials and tensors in anisotropic materials. They depend on extrinsic parameters and are related to the rheological properties of the material.

Types of material responses

Stress will permanently deform a body of material only if the **strength** of the body is exceeded. In simple terms, strength is the maximum differential stress that a material can support under given conditions. Theoretical continuum deformation can be described with three end-members, rheological behaviours: elasticity, viscous flow and plasticity. These three behaviours refer to three ideal rheological models, which are the characteristic relationships between stress, strain and time, exhibited by analogue objects being deformed. In order to understand deformation in rock, it is convenient to examine separately and in combination the three rheological end-members:

- Reversible elastic rheology, at small stresses and strains
- Irreversible, rate dependent **creep** (viscous rheology) which is usually thermally activated
- Rate independent, instantaneous yielding at high stresses (plastic rheology), which is often pressure sensitive but temperature independent.

In this introduction, only the one-dimensional macroscopic behaviour will be discussed.

Food for thoughts

Drop on the floor: (1) a jelly bean (or a gum eraser), (2) a rusk or a cracker, (3) a ball of dough (or soft clay or silly putty) and (4) some honey or heavy syrup. They all are submitted to the same gravity forces and they all follow the same trajectory. Describe their difference when they hit the ground, and relate their behaviour to those introduced up to now.

Elastic deformation

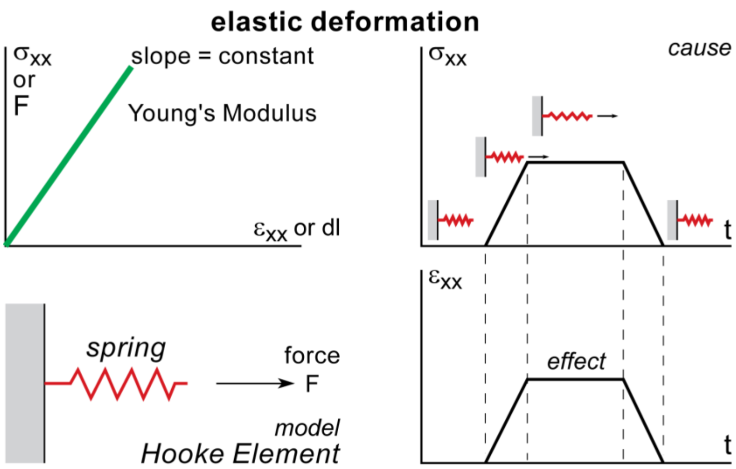
Elastic rheology has wide applications in geodynamics and constitutes a fundament to the plate tectonic theory according to which the lithospheric plates do not internally deform significantly over geological time. The compression or extension of a helical spring (Hookean body) demonstrates the elastic deformation and response.

Definition

Deformation is perfectly **elastic** when straining or unstraining takes place instantaneously once the load is applied or removed, and strain is strictly proportional to stress. An elastic medium deforming instantaneously and reversibly under local stresses has no **memory** of past deformations and stresses. Strain exists only if stress exists. Importantly also, the principal axes of strain must coincide with the principal axes of stress in isotropic materials.

Occurrence in rocks

When a sound wave from an earthquake or an explosion travels through a body of rock, the rock particles are infinitesimally displaced from their equilibrium positions. They return to these positions once the disturbance has passed; there is no **permanent distortion** of the rock. The same kind of temporary and totally **recoverable deformation** occurs if a rock or mineral specimen is loaded axially in the laboratory at relatively low stress (the latter really meaning differential stress) and at low hydrostatic pressures and temperatures. The instant the load is applied, the specimen begins to deform. The ideal relationship between the axial stress and the **longitudinal strain** is linear. Provided stress and strain remain small, the specimen does not break and instantly returns to its original unstrained size and shape once the deforming load is removed.



Modulus of elasticity

The linear relationship between stress and longitudinal strain in tension as well as in compression is written:

$$\sigma = E\,\varepsilon = E(\ell - \ell_0)/\ell_0 \quad (1)$$

This equation is the Hooke’s law where:

σ is the applied stress,

ε is the dimensionless extensional strain, proportional to σ,

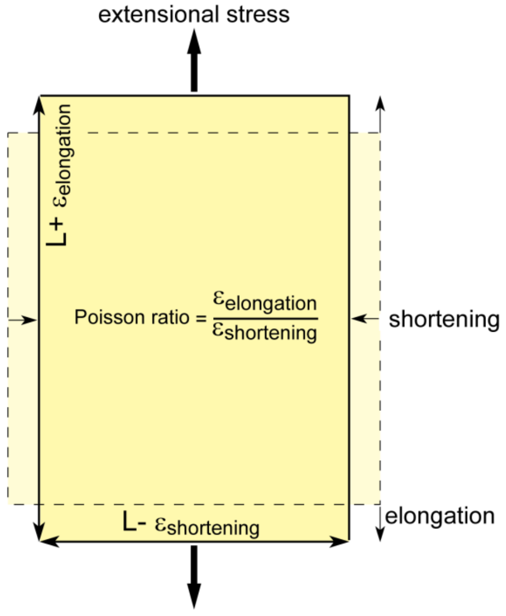
ℓ the deformed-state length, ℓ_0 the original length, and

E is a constant of proportionality (for example the strength of the spring) known as **elasticity modulus** or **Young's modulus**, which has the same dimension as stress because strain is dimensionless.

Note that $d\varepsilon/dt = 0$. There are no time-dependent effects.

Poisson number

The **Poisson ratio**, ν, is used to express the relationship between volume change and stress and refers to the phenomenon that elastic materials extended in one direction are simultaneously shortened along the perpendicular direction.



Definition of the transverse shortening (Poisson effect)
for the infinitesimally small extension of an isotropic, elastic body

This lateral strain effect is called the **Poisson effect**. The dimensionless Poisson's number is the ratio of elastic lateral, transverse shortening of an extended rod to its longitudinal extension.

$$\nu = \varepsilon_{parallel-to-extensional-stress} / \varepsilon_{perpendicular-to-extensional-stress} \qquad (2)$$

It shows how much a core of rock bulges as it is shortened.

Shear modulus

The previous equations were considering one-dimensional, tensile or compressional experiments. If the deformation is by simple shear the constant proportionality G is the **shear modulus** (also called **rigidity modulus,** expressed in Pa) defined as the ratio of shear stress τ to shear strain γ:

$$\tau = G\gamma$$

Bulk modulus

In pressure tests, the rock sample is subjected to controlled hydrostatic pressure. As there is volume strain, length strain and shear strain associated with appropriate stresses, a number of other constants replacing E are defined for isotropic elastic materials.
For a uniform hydrostatic pressure P producing a uniform dilatation, the **bulk modulus** or **incompressibility** K is the ratio of the hydrostatic pressure to that dilatation.

$$P = K\frac{V\text{-}V_0}{V_0} = K\frac{dv}{v}$$

where V and V_0 are the final and initial volumes, respectively.
The inverse of the bulk modulus $k = 1/K$ is the compressibility. The units are Pa.
The four quantities E, ν, G and K are related by the expressions:

$$G = E/\left[2(1+\nu)\right] = \left[3K(1-2\nu)\right]/\left[2(1+\nu)\right]$$

Strain energy

In an ideal elastic body, all the energy introduced during deformation remains available for returning the body to its original state. This stored, **internal strain energy** does not dissipate into heat, which makes elasticity the only thermodynamically reversible rheological behaviour.

Viscous deformation

The intricately folded rocks that exist in some tectonic environments indicate that under certain conditions rocks can undergo large permanent strains without obvious faulting or loss of continuity. Most of the structures of interest to geologists involve strains that are **permanent** and irreversible: There is no **recovery** (i.e. elimination of strain: the rock remains in a strained state) after removal of the deforming stress.

Definition

The **ideally viscous** (**Newtonian)** behaviour is best exhibited by the flow of fluids. The compression or extension of a dashpot (Newtonian body), a porous piston that slides in a cylinder containing a fluid, demonstrates this type of deformation and response. When a force is applied to the piston, it moves. The resistance of the fluid to the piston moving through it represents viscous resistance to flow. When the force is removed the piston does not go back: Deformation is irreversible (non-recoverable) and permanent. The viscous behaviour is said to be dissipative.

Viscosity

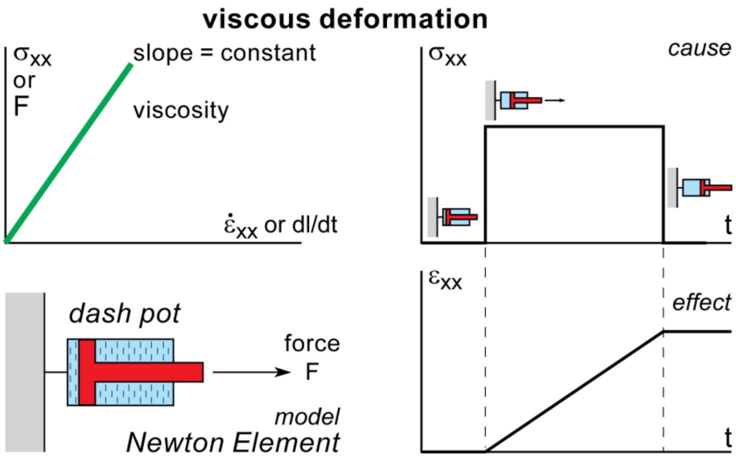
The ideally viscous (Newtonian) material is incompressible. In this material, the strain-rate is proportional to the applied stress:

$$\sigma = \eta.\dot{\varepsilon} \qquad (3)$$

Where: $\dot{\varepsilon}$ is the strain rate (i.e. $d\varepsilon/dt$, the total strain derivative with respect to time) and,

η, the constant of proportionality, is the **viscosity**.

The unit of viscosity has the dimension of stress $[ML^{-1}T^{-2}]$ multiplied by time, therefore $[ML^{-1}T^{-1}]$. It is 1 Pa.s. The bulk viscosity of the mantle is of the order of 10^{21} Pa.s. Typical geological strain rates are 10^{-12} s^{-1} to 10^{-15} s^{-1}.



Equation (3) says that the higher the applied stress, the faster the material will deform. Conversely, a higher flow rate is associated with an increase in the magnitude of shear stress. The total strain is dependent both on the magnitude of the stress and the length of time for which it is applied. Large permanent strains whose amount is a function of time can be achieved. Like for elasticity, strain and stress appear or disappear simultaneously, with the corollary that any deformation along time produces local shear stress.

Note that an ideal viscous fluid has no shear strength and its viscosity is independent of stress.
For anisotropic materials (3) is replaced by a system of nine linear equations.

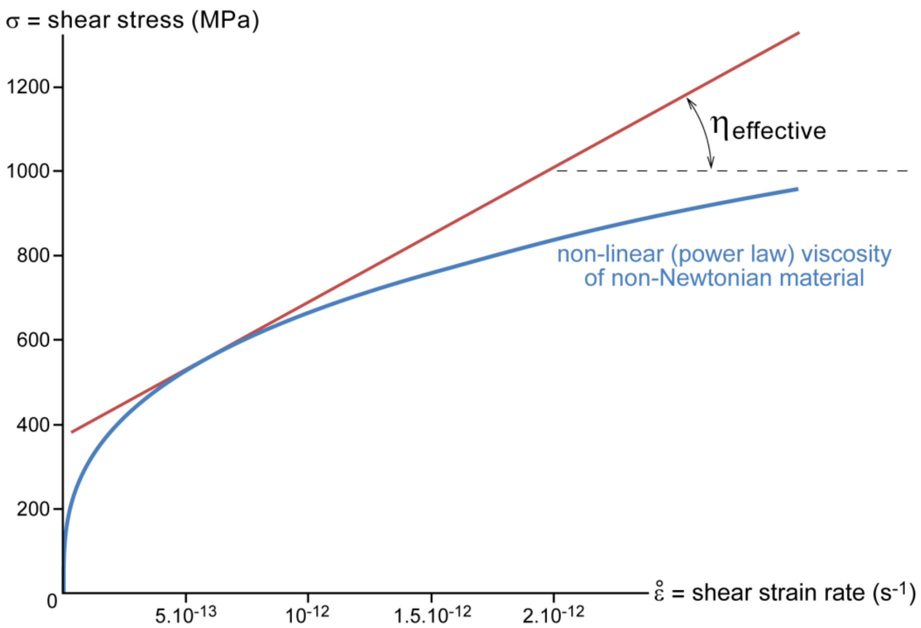
Non-linear behaviour

The linear viscosity is a close approximation to that of real rocks at high temperatures (1000-1500°C) and slow strain rates (10^{-12} to 10^{-14} sec^{-1}). Such physical conditions are found in the lower lithospheric mantle. The viscous behaviour of upper mantle and crustal rocks is complicated by two important facts:

(i) Viscosity is a strong exponential function of temperature (the Arrhenius relationship). Viscosity decreases with temperature.

(ii) The proportionality between stress and strain rate is typically not linear, but governed by a law stating that stress raised to some power is proportional to strain rate. The stress exponent of rocks is typically between 3 and 5, so that the application of a doubled stress results in an 8 to 32 fold increase in strain rate.

The result is that viscosity of rocks in the related stress-strain rate plot is a curve. Because of the non-linear behaviour of rocks, one uses the **effective viscosity**, which is defined by the slope of the tangent to the viscosity curve. The effective viscosity is no material property. It is a description of the viscous behaviour at specified stress, strain rate and temperature.



Non-Newtonian viscosity of olivine at 800°C
and its effective viscosity for a strain rate of $6.10^{-13}.s^{-1}$

Plastic deformation

Plasticity deals with the behaviour of a solid.

Definition

The **ideally plastic** material is a solid that does not deform until a threshold strength = stress (the **yield stress)** σ_c is reached, and incapable of maintaining a stress greater than this critical value σ_c. At the yield stress, permanent and irreversible deformation proceeds continuously and indefinitely under constant stress (as for viscous deformation); it is therefore theoretically possible to get unlimited plastic deformation. The amount of strain, i.e. of plastic flow is a function of time, as long as the yield stress is maintained. Plastic flow on a macroscopic scale can be spatially continuous (uniform) or discontinuous (e.g. faulting). It is a shear strain at constant volume and can only be caused by shear stress.

Yield criterion

Ideal plastic behaviour is rate independent. It assumes that there is no deformation below the yield stress and that during deformation the stress cannot rise above the yield stress, except during acceleration of the deformation. Yet, strain rate is independent of stress. The constitutive equation where stress for flow is a constant is the **Von Mises yield criterion**:

$$\sigma \leq K$$

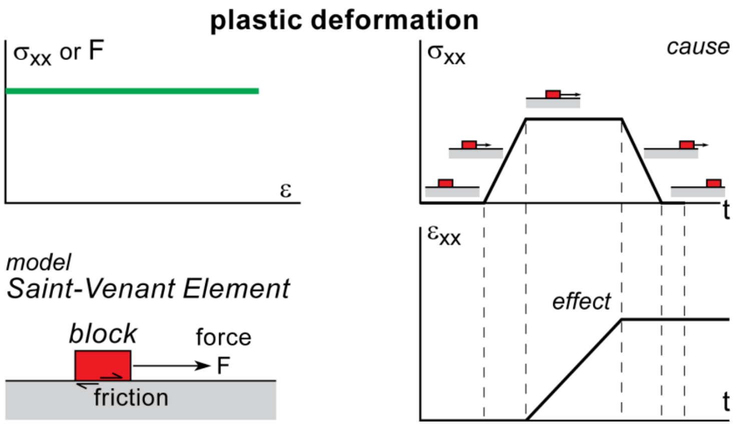
which requires that the magnitude of stress cannot exceed the yield stress $K = \sigma_c$. This particular stress is the characteristic **strength** of the material. The strength is not a constant but a dependent variable; it is a function of the three principal stresses applied and of the temperature, pressure, of the nature of the material, the chemical composition of the adjacent rocks and, finally, of the history of the deformation (i.e. the intermediate steps followed to attend the yield value). Time does not appear in the constitutive equation. Neither strain nor strain rate is related to stress.

Model

The conceptual model to simulate this type of deformation and response is a weight resting on a flat and rough surface (Saint-Venant body). The weight is not displaced as long as the applied force is less than the frictional resistance. At a threshold force the weight begins moving, and a constant force that just overcomes the frictional resistance keeps it moving. When the force is removed or decreases below the threshold magnitude, the weight stays in its new position. The analogy is not really a case of plastic deformation. It describes the relationships between stress, strain (displacement) and time, but the weight remains undeformed. A plastic body is deformed while displaying similar relationships between stress, strain and time. The flow rule is a function of stress:

$$\dot{\varepsilon} = \begin{cases} 0 & \text{if} \quad \sigma \leq K \quad \text{and} \quad \dot{\sigma} = 0 \\ \dot{\Lambda}.f(\sigma) & \text{if} \quad \sigma = K \quad \text{and} \quad \dot{\sigma} = 0 \end{cases}$$

In which $\dot{\Lambda}$ is a positive and indeterminate proportionality factor. This indeterminacy, along with the existence of K, distinguish perfectly plastic from viscous materials.



Note that the stress does not determine the strain rate, but the stress and strain rates behave in an analogous manner. The ideally plastic material displays another characteristic kind of deformation: strain only takes place in localised regions where the critical value of stress is reached.

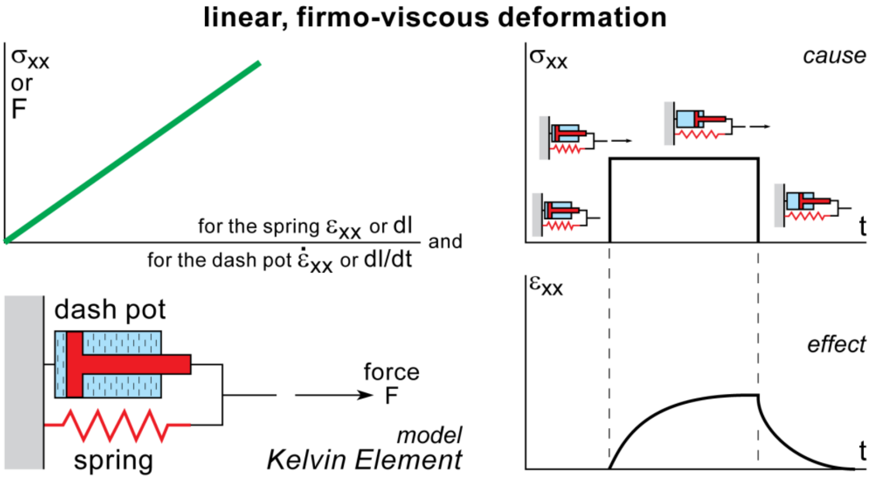
Viscoelastic deformation

Real rocks, whether shallow or deep in the Earth, combine the properties of ideal viscous, plastic and/or elastic bodies. Their strain has elastic properties on the time scale of seismic waves, and

plastic or viscous components or any combination of these behaviours on the long, plate-tectonics time scale. A material that combines viscous and elastic characteristics is **viscoelastic**.
Viscoelastic behaviours are useful in modelling the response of the earth's lithosphere, which exhibits elastic deformation over a short time-scale but gradually flows on long terms. Mechanical models of such materials are represented by a spring and a dashpot arranged in series or in parallel.

Firmo-viscous (i.e. strong-viscous) behaviour (Kelvin body)

A spring and a dashpot in parallel (known as the Kelvin body) simulate the strong-viscous (firmo-viscous) deformation.



When the force is applied both the spring and the dashpot move simultaneously. The deformation (i.e. the displacement) is the same for both. However, the dashpot and spring stresses are in parallel and thus the total stress is the sum of the stress in the spring and the stress in the dashpot. From adding equations (1) and (3), the Kelvin model has the constitutive equation:

$$\sigma = E\varepsilon + \eta\dot{\varepsilon}$$

Note that: If $E \to 0$ (the spring has zero stiffness), the flow is viscous.
If $\eta \to 0$ (i.e. for low viscosities) the material is elastic.

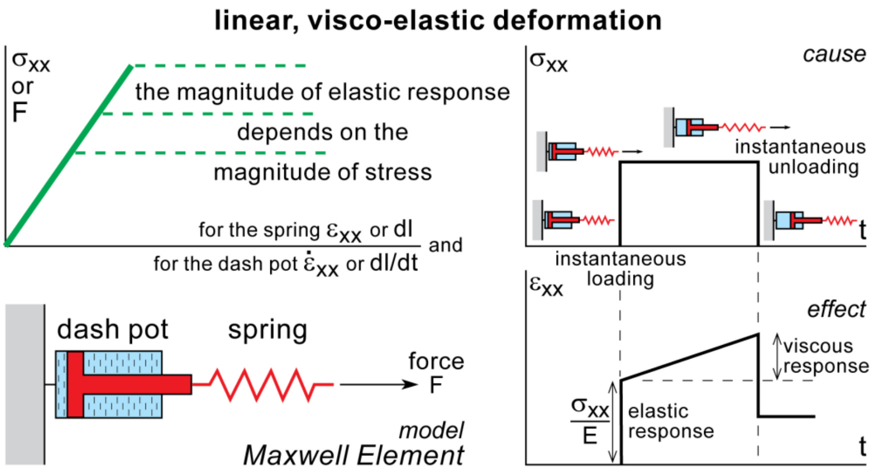
Because of the parallel arrangement of the dashpot and the spring, the Kelvin body is characterised by complete recovery of its geometry when an applied force is removed. However, the dashpot retards elastic shortening of the spring. Stress gradually decreases until all of the strain is recovered. When the force is removed, the strain does not immediately disappear. This is known as **elastic after-effect.**
Note that a suddenly applied stress will induce no instantaneous strain because of the dashpot in parallel to the spring.
Some unconsolidated geological materials approximate this behaviour.

Visco-elastic behaviour (Maxwell body)

A visco-elastic material basically obeys the viscous law (i.e. strain is a function of time), but behaves elastically at the instant of stress application and for stress of short duration.

Rheology

A spring and a dashpot arranged in series (Maxwell body) represent this type of behaviour. This arrangement indicates that the spring and the dashpot take up the same stress (the forces and the stress they represent are equal in the spring and in the dashpot) but the total strain (as well as the strain rate) is the sum of the spring deformation and the dashpot deformation.



When a Maxwell body is subjected to a stress, the linear strain rate $\dot{\varepsilon}$ is composed of two parts: (1) The spring is instantaneously and elastically extended at a strain rate directly proportional to the stress rate $\dot{\sigma}$; (2) the dashpot responds only to the instantaneous level of stress, and moves at a constant rate controlled by the viscosity for as long as the force is applied. Using equations (1) and (3), the corresponding constitutive equation is:

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta} \qquad (4)$$

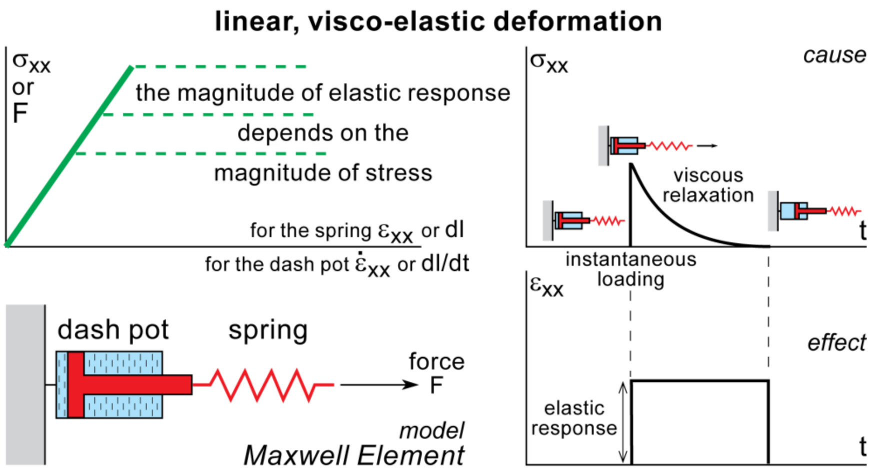
Note that: If $E \rightarrow \infty$ (the spring is rigid), the material behaves like a viscous material for long-term loads.

If $\eta \rightarrow \infty$ (i.e. for high viscosities) the material behaves like an elastic material for loads of short duration.

Note also that, in contrast to the Kelvin body, a suddenly applied stress will induce an instantaneous elastic strain because the spring is free to respond.

Relaxation time

For simulating a permanent deformation, the spring is extended and then held at a given extension (i.e. the material is under constant imposed strain). The spring deforms instantaneously and elastically as soon as a constant force F is applied. The spring instantaneously stores energy gradually converted into permanent viscous deformation as the dashpot moves at a constant rate through the fluid until the spring has returned to its unstressed length. Provided no force is still applied, the spring only may recover its original length, but the dashpot maintains some irreversible flow. The viscosity of the liquid retards elastic recovery, which is rapid at first but decays as the tension in the spring decays. When the application of force F stops, stress neither vanishes nor persists; instead it dissipates exponentially. This is called **stress relaxation.**



The time taken for the stress to decay to e^{-1} of its initial value is known as the **Maxwell relaxation time** t_M. e is the base of natural logarithms. The relaxation time can be obtained by dividing viscosity by the shear modulus:

$$t_M = \eta / G$$

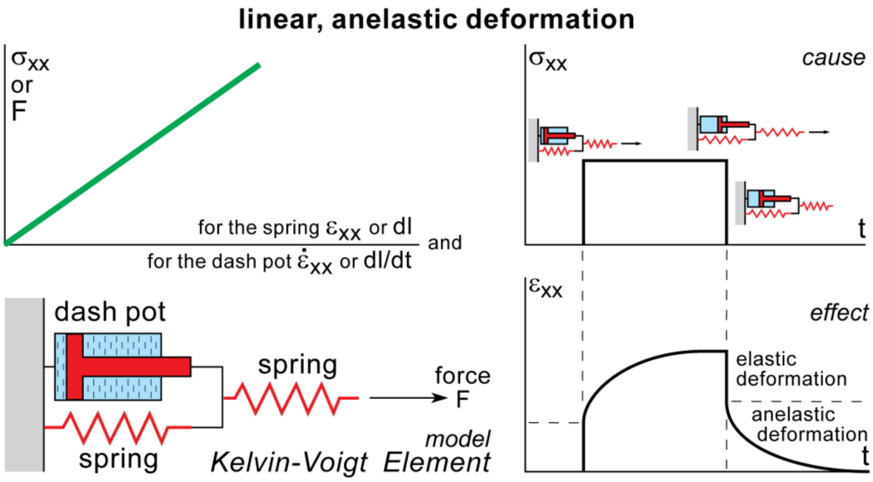
Exercise

Calculate the relaxation time for olivine-dominated rocks (Earth mantle): G= 10^{12} dyn.cm^{-2} and $\eta = 10^{22}$ poise.(answer, about 10^{10} s, ie. about 300 years).

Anelastic deformation (Kelvin-Voigt body)

Although the pre-seismic strain in the upper crust is mostly recoverable, the complete response of rocks is not instantaneous. This type of non-perfectly elastic behaviour where unstraining is recoverable but not instantaneous (time dependent) is called **anelastic**.

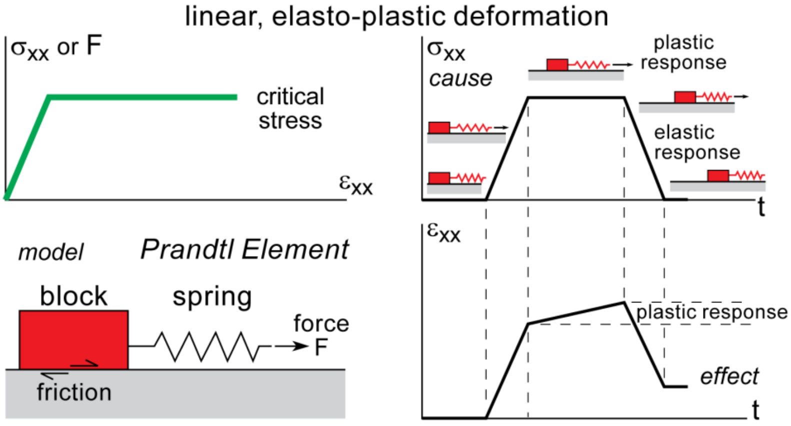
The mechanical model of a standard linear solid comprising a spring (a Hookean body) in series with a unit of parallel dashpot and spring (a firmo-viscous Kelvin body) represents the anelastic behaviour. A plot of strain against time illustrates this for a specimen loaded axially. When the force is applied there is an instantaneous elastic response due to the first spring. Stretching of the second spring is inhibited by the viscosity of the fluid in the dashpot with which it is in parallel. Later flow comprises elastic deformation delayed by the action of viscosity. When the force is removed, there is again an instantaneous elastic response and then the strain decays asymptotically back to zero.



Anelasticity is of great importance in many rock mechanics problems associated with mining, tunnelling and quarrying. Anelastic behaviour is associated with reversible, time-dependent slipping along grain boundaries (internal friction). This strain response absorbs energy from seismic and any sound waves moving through a rock. The magnitude of this **attenuation** depends on environmental parameters such as temperature, pressure, and the frequency of the propagating wave, for example as they pass through parts of the upper mantle of the earth. This behaviour is also termed recoverable **transient creep.** The anelastic motion of a fluid phase, either as a melt or a saturated hydrous fluid, will give rise to energy losses from sound waves propagating through a rock and to a parallel decrease in the velocity of the waves through the rock. This phenomenon has been widely postulated as the source of the upper mantle low velocity zone.

Elasto-plastic behaviour (Prandtl body)

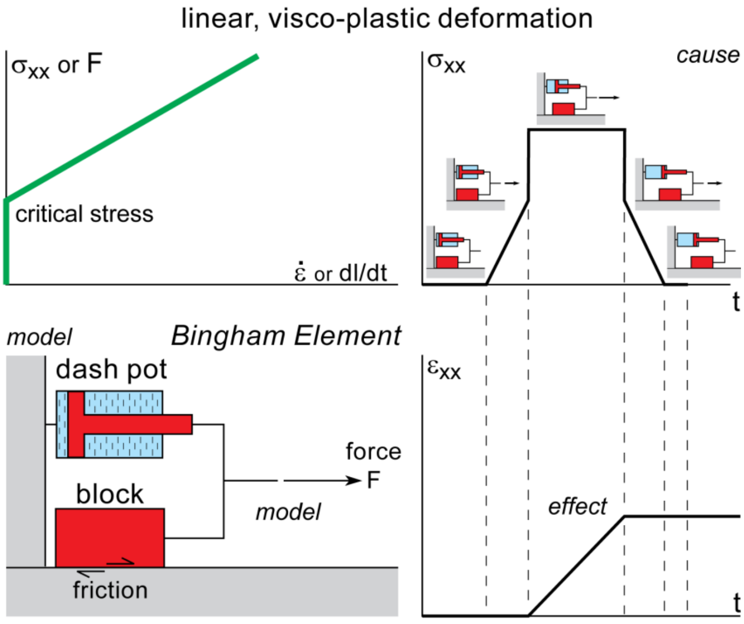
The **elasto-plastic** deformation is simulated by the series arrangement of a spring and a weight (the Prandtl body).



Stress below the yield strength first stretches the spring. Then the weight is pulled at yield stress close to the frictional resistance of the weight, and comes to rest in a new position where the stress state is the same as in the initial state.

Viscoplastic deformation

To eliminate the indeterminacy on $\dot{\Lambda}$ in the flow rule of perfectly plastic materials, one must introduce hardening in the model. The conceptual model (Bingham body) to simulate this type of deformation and response is a weight resting on a flat and rough surface (Saint-Venant body) in parallel with a linear viscous dashpot (Newton body).



If the stress is smaller than the yield stress of the weight, the model is rigid. At the yield stress, an overstress $(\sigma - K)$ is exerted on the dashpot. Accordingly, the flow rule of a viscoplastic material is:

$$\dot{\varepsilon} = \begin{cases} 0 & \text{if} \quad \sigma < K \\ (\sigma - K)/\eta & \text{if} \quad \sigma \geq K \end{cases}$$

Visco-elasto-plastic deformation

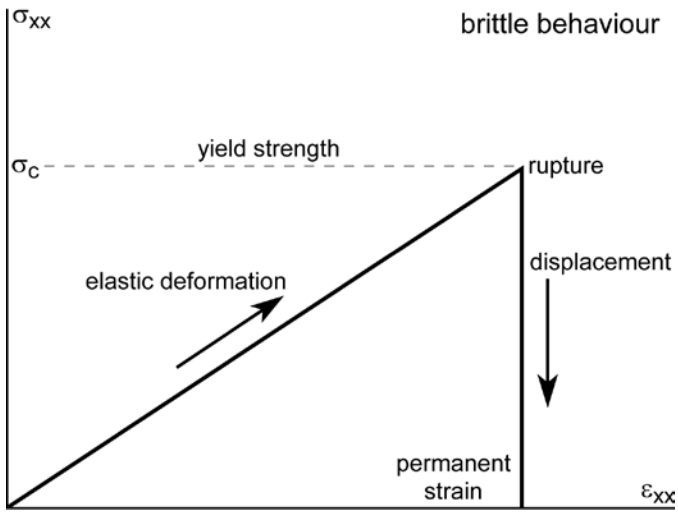
If a viscoelastic material has also a yield stress, the behaviour is **visco-elasto-plastic**. Materials that behave in this manner are called Bingham materials. They are represented by a spring, a dashpot and a friction block on a raw surface (Hookean + Newtonian + Saint-Venant bodies) arranged in series or in parallel. The deformation is distributed between the mechanisms according to their material properties.
On applying a load there immediately is elastic deformation (from the spring). If the applied force is greater than friction resistance of the weight, this element will come into play together with the dashpot and visco-plastic flow will take place at a constant rate. The strain/time curve is similar in shape as for an elasto-viscous behaviour in both loading and unloading phases. The main difference is that flow begins only after the force applied on the Bingham material has exceeded the frictional resistance of the Saint-Venant body, which simulates the limit of plasticity. Below this limit, only elastic deformation can take place, whereas flow takes place no matter how low the load is on the elasto-viscous body. In this respect, visco-elasto-plastic models portray the properties of rocks more accurately than other models. However, the model does not allow effects of relaxation.

Brittle / ductile behaviour

Rocks like many usual materials are reacting in an elastic manner only for small strains. When the yield stress is attended, there are two possible behaviours:
– rupture, when the continuity of the deformation is lost. This behaviour is **brittle**, where elastic deformation leads to **failure** before plastic deformation and the material loses cohesion through the development of **fractures** or **faults**.
– a plastic, irreversible flow (**creep**) while, apparently, the continuity holds. The deformation is quite similar to the usual viscous flow, but it can be observed only when the yield stress is attended. This behaviour is **ductile**.

Brittle behaviour

The maximum stress a rock can withstand before beginning to deform permanently (inelastic behaviour) is its **yield point,** or **elastic limit**.



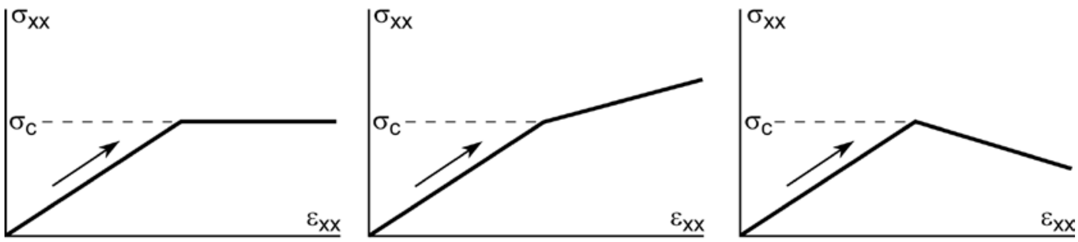
At this stress level, and at low confining pressure and low temperature, most rocks and minerals break into fragments. Localised deformation at the yield stress is permanent; therefore, the brittle behaviour yields a plastic deformation. With this definition, brittle behaviour refers to a state of stress at which rupture occurs.
The brittle mode of deformation includes both fracturing and sliding, hence governs the development of joints and faults. Open fractures are generally parallel to the axis of loading and there is no offset parallel to the fracture surface. Faults are inclined to the axis of loading, and show

a localised offset parallel to their surface. **Cataclastic flow** is achieved by distributed fracturing and the relative movement of rock fragments. The mechanical properties of rocks deforming in the brittle regime are not very sensitive to temperature, but they are quite sensitive to strain rate and pressure. Indeed, friction critically depends on the pressure acting across planes and the fracture strength increases with depth. Therefore, the fracture strength of rocks at the Earth's surface is the lowest and is controlled by the failure criteria only, but it increases with depth due to increasing **lithostatic pressure**. In reality, friction is at any depth rather controlled by the **effective pressure**, the difference between the lithostatic pressure and the **pore fluid pressure** acting against it.

Ductile behaviour

The **ductile** behaviour is a non-mechanistic, phenomenological term to describe the non-brittle modes of deformation; rocks exceeding their yield point deform by distributing the permanent strain in a smoothly varying manner throughout the deformed mass without any marked discontinuity. The term **ductility** is used in geology to indicate the capacity (% of strain) of a rock to undergo permanent deformation without the development of macroscopic fractures.

Ductile flow commonly involves deformation of individual grains by a number of solid state deformation mechanisms such as crystallographic slip, twinning, or other processes in which atomic diffusion plays a part. A set of conditions (relative heat, pressure, time, etc.) must be met before the rock deforms. The ductile behaviour is dominantly temperature-dependent and prevails in the deeper crustal and lithospheric levels or in regions with a high thermal gradient. The stress magnitude, strain rate and the mineral composition of the rock medium are other important controlling parameters. The effect of increased temperature and decreased strain rate is to promote **thermally activated** processes such as crystal slip and atomic diffusion. Creep is the time-dependent flow of solid material under constant stress. Creep can be by viscous or by plastic flow. **Plasticity** refers to the property of crystals to deform permanently by slip along lattice planes, which is not necessarily proportional to time.



Flow of ductile material

Provided the temperatures are sufficient, rocks begin to creep at low stresses. It means that ductile parts of the lithosphere are very weak in comparison to the elastic-rigid lithospheric parts and may be treated as viscous at long time scales.

Brittle-ductile transition

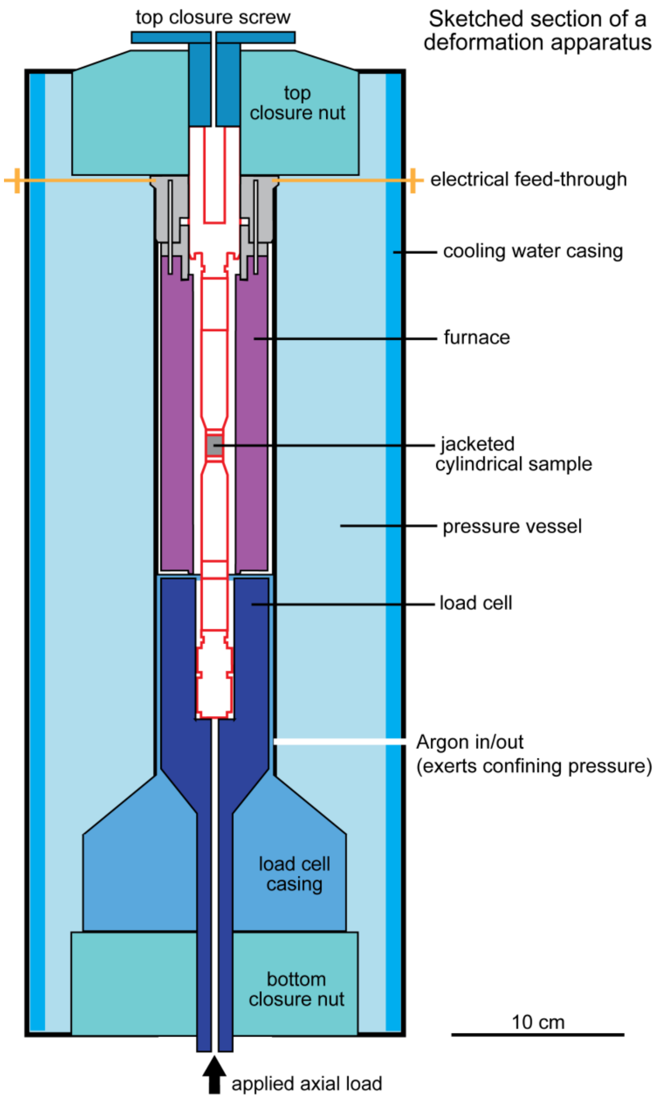
The effect of increased hydrostatic pressure is to inhibit fracturing and cracking. As the hydrostatic pressure is increased the behaviour of rocks passes through a transition from brittle to ductile behaviour for each particular type of rock. This is called the **brittle-ductile transition.** Its depth generally represents the lower limit of most crustal seismicity. This transition is not sharp, nor is it a consistent depth or temperature. It is a function not only of hydrostatic pressure, but of temperature and of strain rate as well. In general, the lower the temperature and hydrostatic pressure, and the higher the strain rate, the more likely is a rock to behave in a brittle manner. On the other hand, the higher the temperature and hydrostatic pressure and the lower the strain rate (or the longer time the stress is applied), the more likely is a rock to behave in a ductile manner. In effect, there is a broad transition between brittle and ductile behaviours, where "semi-brittle" or "semi-ductile deformation involves a mixture of brittle and ductile processes on the microscale.

STRESS AND STRAIN IN ROCKS

Rocks and minerals are natural solid materials. To know as much as possible about the behaviour of rocks and relate strain and strain rates with stress, one need to experimentally deform rocks under varied and controlled conditions of temperature, pressure, fluids and time.

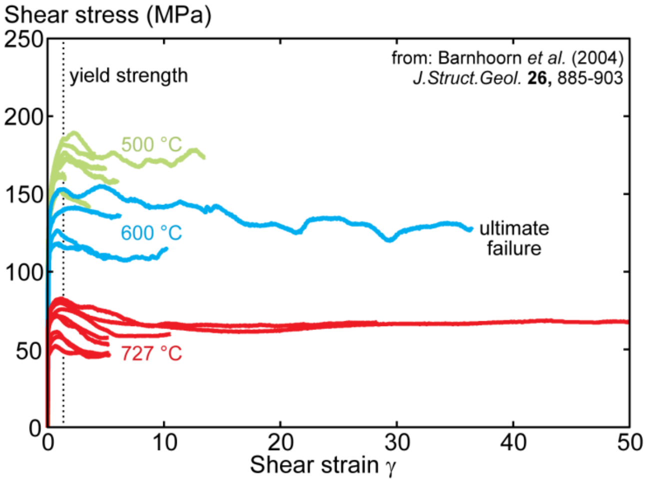
Most mechanical tests consist in compressing a small cylindrical rock sample along its axis with a piston while exerting a confining pressure to all sides of the sample with a pressurised fluid that surrounds the specimen in a pressure chamber. These pressures simulate both a realistic force of compression and the confining pressure to which materials deep in the crust are subjected, i.e. the weight of the overlying rock. The fluid transmits a uniform confining pressure to the specimen through an impermeable, flexible sleeve called jacket, usually made of copper. This is known as **triaxial test** because it allows predetermining stress to be applied along each of the three principal axes, with the main compression oriented parallel to the cylinder long axis. However, two of the principal stresses are equal.

The axial force applied by pistons to the ends of the test cylinder result in either maximum (compression) or minimum (tensile) stress along the axis of the cylinder, depending on the relative magnitude of the confining pressure and on the axial force. The difference between the axial stress and the confining pressure is the **differential stress** $(\sigma_1 - \sigma_3)$. Both of the remaining principal stresses are equal to the confining pressure. By varying any or all the pressures (axial load, confining and pore pressures), one can obtain different stress configurations.



Constant Strain-Rate Experiments - Effects of variation in stress

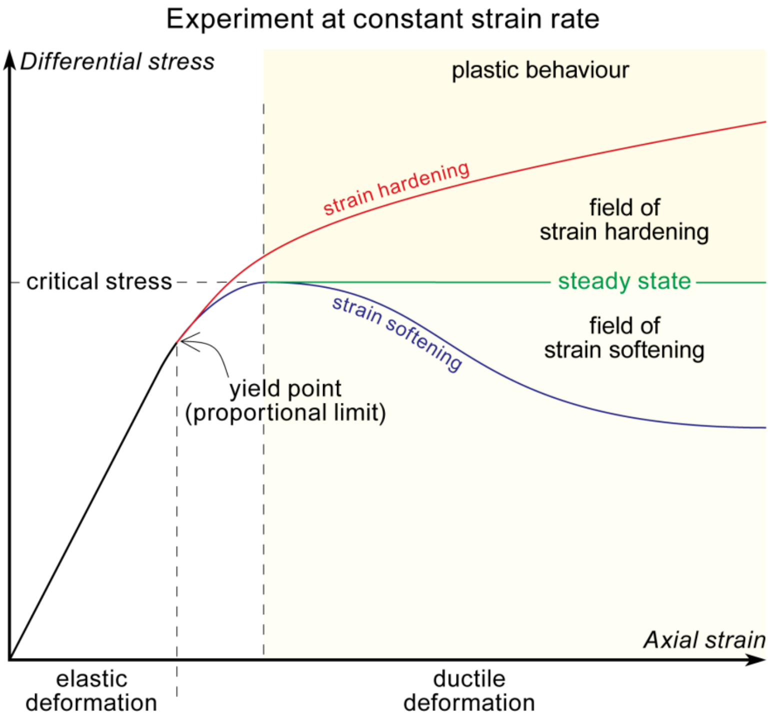
In constant strain rate experiments, the piston of the machine moves at a constant rate throughout the experiment. The specimen deforms at a constant rate. In order to maintain this rate, the stress is allowed to vary and this is recorded. Results are presented graphically on a load /ensuing strain (stress/strain) diagram.



Rheological data from deformation experiments on Carrara marble under constant shear strain rate

Elastic limit

Starting from the origin, i.e. at the onset of experiment, the strain-time curve begins with a straight linear segment, along which the strain increment is proportional to the stress increment. If the load is removed, the sample will recover its initial length.



Idealised stress-strain curve obtained by experimental rock deformation
adapted from Tullis & Tullis (1986) *Geophysical Monograph* **36**, 297-324

The linear curve documents that material first deforms elastically. The stress corresponding to the end of this linear section is called **proportionality limit**. For slightly larger values of differential stress, a change in slope of the strain/stress diagram indicates the beginning of permanent deformation. The material has passed through an **elastic limit** where the stress is no longer proportional to strain. The point where the permanent strain begins is the **yield point** (corresponding to a **yield stress**).

Strain hardening

Continuing an experiment at low temperatures, the slope of the curve diminishes beyond the yield point. In many materials, an ever-increasing stress (slower than in the elastic domain) is required for deformation to increase from the yield point onward. The material exhibits essentially viscous behaviour while undergoing permanent deformation. This effect is the **strain hardening**.
Strain hardening is responsible for another phenomenon. If load is removed, strain due to elastic deformation is recovered but strain due to plastic deformation remains. Unloading does not interrupt the stress-strain curve. If a load is applied again to the same sample, initial deformation is again elastic, but the elastic limit is higher than the first yield stress. The yield strength of the specimen has increased because the original fabric, thus properties of the sample, has been modified by permanent plastic deformation.
Strain hardening can be suppressed by prolonged but moderate heating (**recovery**) or by intense heating that induces a total recrystallization of the material (**annealing**).

Steady state

The yield strength is a variable that depends on the past plastic deformation of the sample. At high temperatures or slow strain rates, the curve becomes horizontal from the yield point onward: no strain hardening accompanies permanent deformation, and conditions approach **steady state**. Further strain increments take place with little or no increase in stress. This is the essential characteristic of plastic strain.

Strain softening

Above a second critical value of stress known as the **ultimate strength**, the curve descends to a failure point. While the curve falls the material exhibits accelerated viscous flow leading to rupture at the **failure stress.** In other words, the material becomes so weak that it needs lower stresses to further deform. This behaviour is called **strain softening**.

Shear fracture criteria

At rupture, the normal stress σ_N and the shear stress σ_S acting on a plane are related by an equation of the form:

$$\sigma_S = f\left(\sigma_N\right) \qquad (5)$$

and experiments have shown that, for materials with no cohesion strength such as soils, the relationship is

$$\sigma_S = \sigma_N \tan\phi$$

where ϕ is known as the **angle of internal friction** (it is in this linear equation the slope of a line). The term internal friction describes a material property of slip resistance along the fracture.

Coulomb criterion

Coulomb proposed in 1773 that shear fracture occurs when the shear stress on a potential fault plane reaches and overcomes a critical value. The relationship (5) becomes the **Coulomb failure criterion:**

$$\sigma_S = c + \mu.\sigma_N \quad (6)$$

where c is a material constant known as the **cohesion** or the **shear strength**;

μ is another material constant known as the **coefficient of internal friction** equivalent to the term $\tan\phi$ seen for soils.

Equation (6) assumes that shear fracture in solids involves two factors: breaking cohesive bonds between particles of intact rock (the c term), together with frictional sliding (the term μ, proportional to the normal compressive stress σ_N acting across the potential fracture plane). This physical interpretation predicts a linear increase of rock strength with normal stresses acting on the rock and fits reasonably well much experimental data. Experiments provide cohesive strength of the order of 10-20 MPa for most sedimentary rocks and 50 MPa for crystalline rocks. The average angle of internal friction is 30°.
The Coulomb criterion predicts that shear fractures form at less than 45° to σ_1 because of the positive slope of the shearing resistance curve and the symmetrical shape of the shear stress curve. This criterion, also termed Mohr-Coulomb and Navier-Coulomb yield criterion, governs the creation of a new fracture.

Byerlee's law

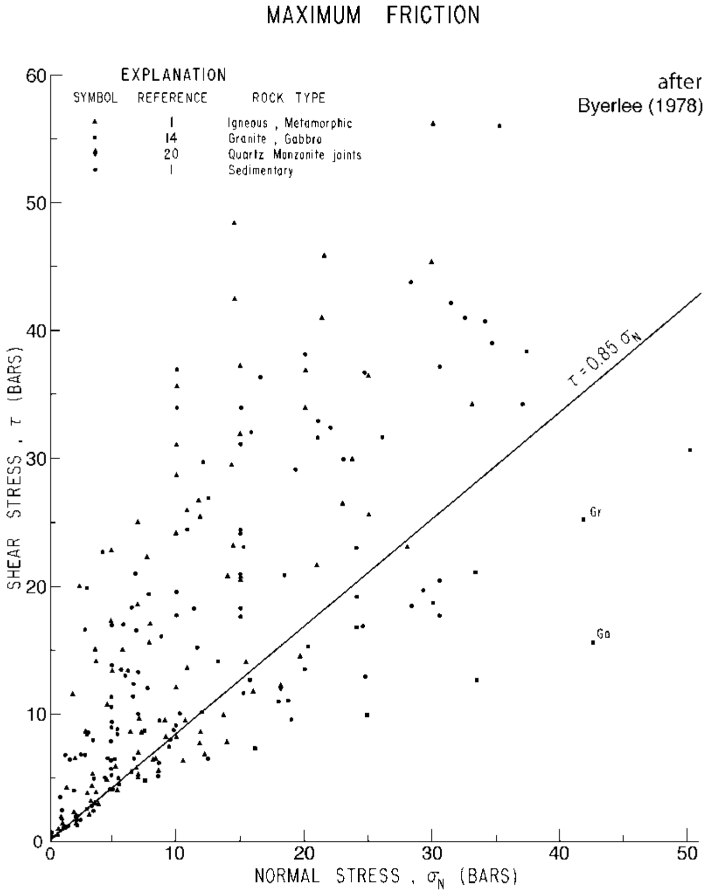
The frictional strength on fault planes is generally constant. The coefficient of internal friction μ on existing fractures in consolidated rocks determines which shear stress is required to cause further movement on the fault planes:

$$\sigma_S = \mu\,\sigma_N \quad (7)$$

This equation, usually valid when two rough surfaces are in contact, is known as Amonton's law. A direct consequence of this law (and the Coulomb criterion) is that the shear stress required for sliding is independent of the surface contact area and increases with normal stress, therefore with confining pressure. The parameter μ, in this general sense, is also referred to as the **coefficient of static friction**.
James Byerlee, an American geophysicist, compiled experimentally determined values of the shear stress required for frictional sliding on pre-cut fault surfaces in a wide range of rock types. He found two best-fit lines that depend on the confining pressure. For confining pressures corresponding to shallow crustal depths (up to $200\,\text{MPa} \approx 8\,\text{km}$), μ = 0.85 and equation (7) becomes:

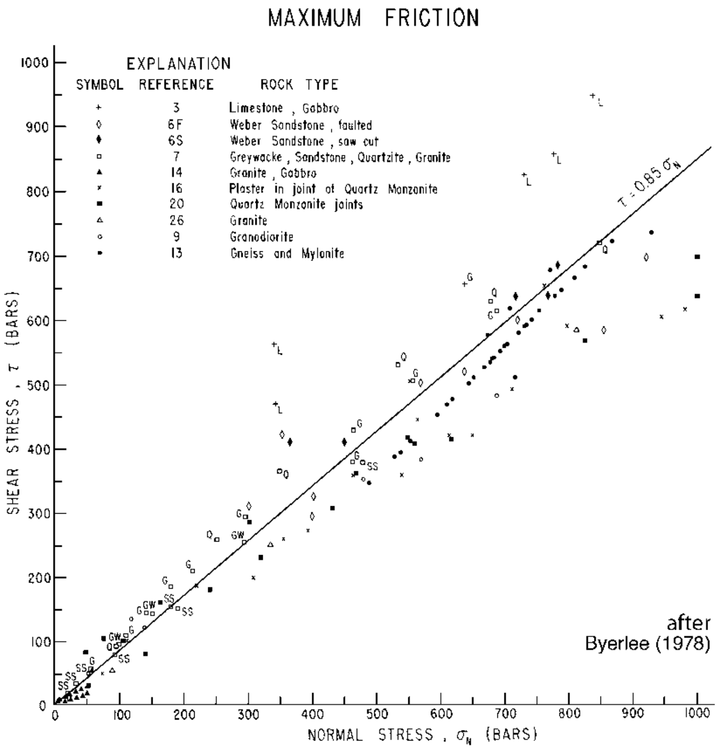
$$\sigma_S = 0.85\,\sigma_N \quad (8)$$

Two diagrams actually illustrate this linear function. The first one refers to very low (<5 MPa) normal stress conditions.



Shear stress plotted as a function of normal stress at the maximum friction for a variety of rock types at normal stresses up to 50 bars.

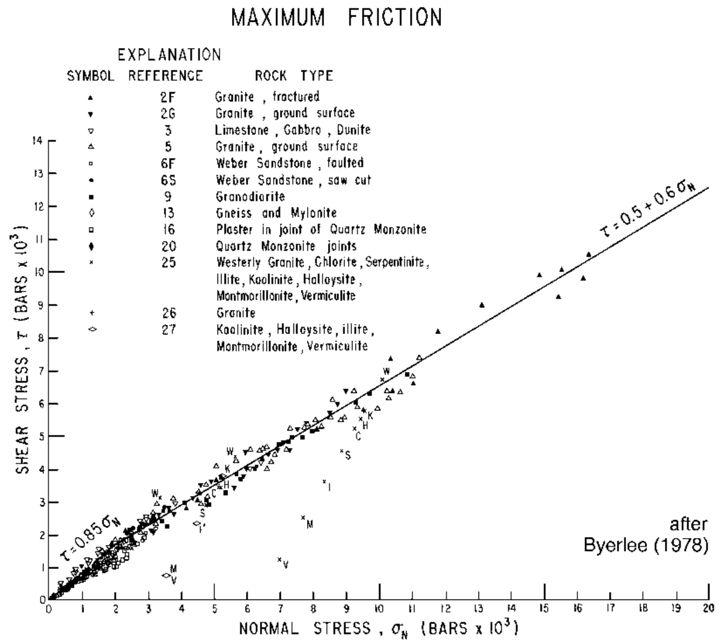
The second one fits laboratory results generated under higher (up to 100 MPa) normal stress conditions.



Shear stress plotted as a function of normal stress at the maximum friction for a variety of rock types at normal stresses to 1000 bars.

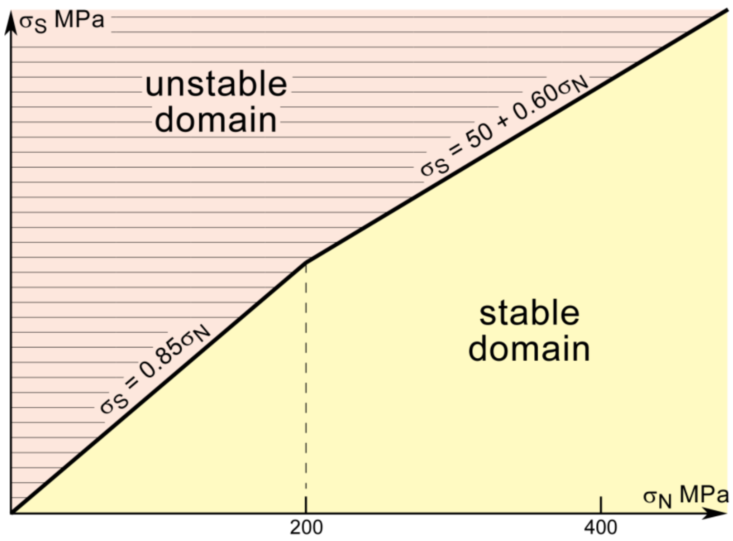
The large scatter of data point under very low normal stress reflects surface roughness, the area of contact of the asperities, less influent under higher confining pressure because the latter prevents dilatancy of the shear fracture, hence unlocking of interwoven surface irregularities. Instead, shearing and smearing of asperities tends to stabilize frictional properties.
For confining pressures between 200 and 2000 MPa, the frictional strength of pre-cut rocks is better described by including a "cohesion-like" parameter:

$$\sigma_S = 50\text{MPa} + 0.6\,\sigma_N \tag{9}$$



Shear stress plotted as a function of normal stress at the maximum friction for a variety of rock types at normal stresses to 20 kb.

The Byerlee's law refers to equations (8) and (9), together. They are empirical and indicate that the shear stress required to activate frictional slip along a pre-existing fracture surface is largely insensitive to the composition of the rock. These laws seem to be valid for normal stresses up to 1500 MPa and temperatures < 400°C, which allows defining a lower boundary to stresses acting in the brittle lithosphere.

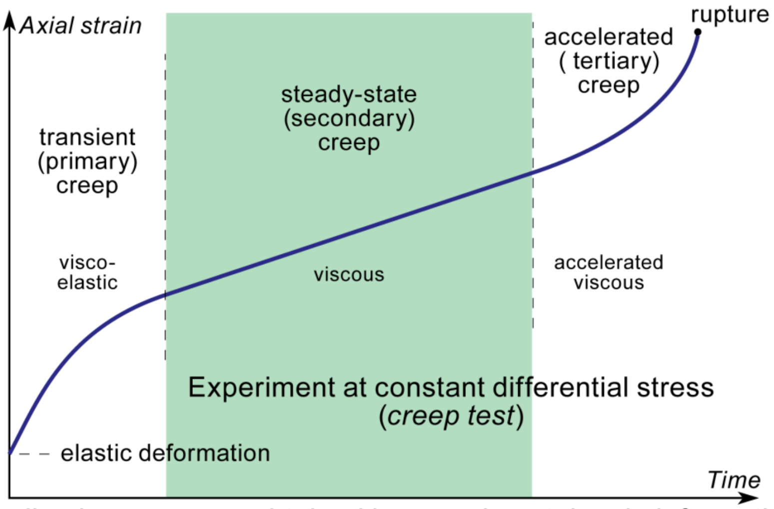


Byerlee's law for pre-existing fault surfaces

Ductile flow

Deformation experiments lasting for days at constant differential stress generally involve the very slow, continuous and plastic creep of the sample. The resulting strain / time curve generally obtained can be divided, after the initial elastic response, into three regions representing decelerating, steady (constant rate) and accelerating creep:

- In the first region, the slope of the curve (i.e. the strain rate) decreases continuously; this decelerating behaviour is called **primary** or **transient creep**; it is logarithmic because the total strain increases with the logarithm of time. The phenomenon of decreasing creep rate at constant stress is called **cold working,** a strain hardening because the material becomes less ductile with increasing strain.



Idealised creep curve obtained by experimental rock deformation
adapted from Tullis & Tullis (1986) *Geophysical Monograph* **36**, 297-324

- In the second, usually largest regions of creep curves, the slope (strain rate) is constant and residual strain is irrecoverable; this flow behaviour is called **secondary** or **steady-state creep**. Even with a constant stress steady-state creep could continue indefinitely. It may represent the long-term deformation processes that occur within the Earth over long times and without failure. Therefore it is the part of the experiment that interests most geologists. The material behaves like a viscous, yet non-Newtonian fluid. As a first order but approximate explanation, steady state creep results from recovery processes (mostly thermal softening) balancing strain hardening as it appears.

- In the third region, not always observed, the strain rate increases exponentially until **rupture** of the specimen. Accelerating flow is mainly caused by the spread of microfractures or slip surfaces through the rock in such a way that they link up (accumulating damage) to form continuous pervasive cracks causing loss of cohesion, and failure; this is called **tertiary** or **accelerating creep**.

Steady-state creep function

During steady-state creep, the differential stress $(\sigma_1 - \sigma_3)$ is related to the strain rate $\dot{\varepsilon}$ by the non-linear, empirical equation:

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = A(\sigma_1 - \sigma_3)^n \exp\frac{-Q}{RT} \qquad (10)$$

where T is the absolute temperature
R is the gas constant

n = an experimentally determined constant depending on the material; it commonly varies between 3 and 5

A is also a material, constant parameter known as the **frequency factor**

Q is the **thermal activation,** a constant that must be determined experimentally. Q has a unit of kcal (or Joules) / mole.

Equation (10) implies that viscosity decreases exponentially with temperature.

Effects of time and strain-rate

If n = 1 in equation (10), then the stress is proportional to the strain rate and the material behaves like a Newtonian, perfectly viscous fluid. The viscosity of rocks with n>1 is characteristically dependent on the strain-rate. The strain rate $\dot{\varepsilon}$ is related to the applied stress by a power law, and equation (3) becomes:

$$\sigma^n = \mathbf{A}\dot{\varepsilon} \tag{11}$$

with $\mathbf{A} = A \exp\frac{-Q}{RT}$ of equation (10), a function of material properties, pressure and temperature.

The proportionality between strain rate and stress is non-linear. For this behaviour, only an effective viscosity can be defined according to:

$$\eta_{eff} = \sigma/\dot{\varepsilon} = \mathbf{A}^{1/n}\dot{\varepsilon}^{(1/n)-1} \tag{12}$$

Effective viscosity is not a material property but a convenient description of rheological properties under known conditions of pressure, temperature, stress and strain rate. It results from equation (12), a small increase in the strain-rate results in a rapid decrease of the effective viscosity, provided $n > 1$. For this reason, the effective viscosity is also called **stress dependent** or **strain rate-dependent** viscosity.

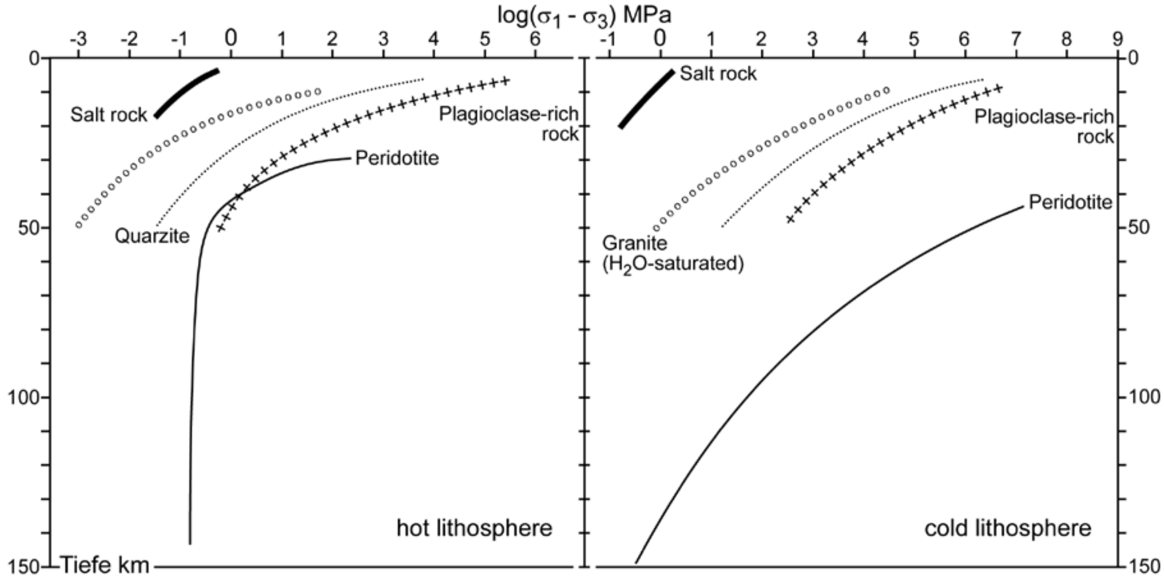
The steady-state rheology of rocks is of great importance to establish constitutive relationships involving stress, strain-rate and intrinsic flow parameters. One particular strain rate yields only one particular deviatoric stress in steady state. A power law with n=3 is often used to represent mantle rocks.

Plastic properties of rocks

The rheology of polymineralic rocks is controlled by the behaviour of the weakest mineral phase, on condition that it constitutes an interconnecting network in a rock, commonly above 30 volume %.

Effect of temperature

Increasing the temperature under identical stress conditions decreases the yield stress, which enlarges the field of ductile, permanent deformation at the expense of the elastic and failure fields. Consequently the material shows an increase in ductility (i.e. the % of strain a rock can take without fracturing at a macroscopic scale). For example, the yield stress of marbles at 800°C is about one-sixth of its value at room temperature. Moreover, the rate at which the material deforms under a given applied stress is increased. A simplified explanation is that heat weakens the bonds between the atoms without breaking them. The temperature effect is well known by blacksmiths who heat metals to fashion objects out of a ductile material that does not break. Similarly, the Earth's internal heat renders brittle rocks more ductile.



Creep strength of various rocks (constant strain rate $10^{-14}s^{-1}$)
after Ranalli & Murphy (1987) *Tectonophysics* **132(4)**, 281-295

These observations are consistent with the geological observation of metamorphic rocks deformed at elevated temperature and pressure. These exhibit much more ductile types of deformation than do the equivalent rocks at the surface.

Effect of hydrostatic (confining) pressure

Variations in **confining pressure** are experimentally introduced to reproduce pressures deep in the crust and mantle.

Rocks deeply buried in the crust are subjected to the **lithostatic pressure** (the vertical normal stress σ_{zz}) which can be assumed to be effectively hydrostatic (i.e. equal in all directions), and simply related to the thickness and mean density of the overlying column of rock. Accordingly, the lithostatic pressure is generally calculated by a simple integration:

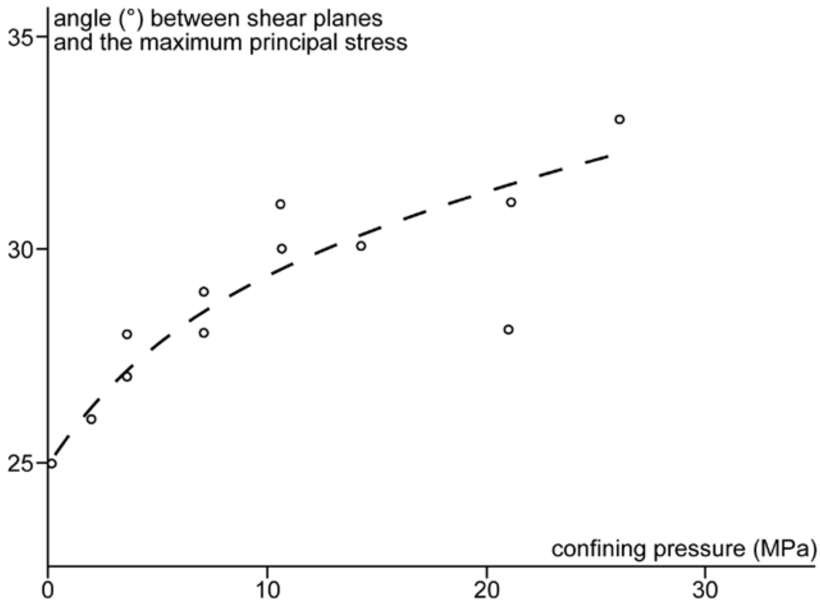
$$P_{lith} = g\int_0^z \rho_{lith}(z).dz \qquad (13)$$

with ρ rock densities, g the gravitational acceleration and z the depth.

The pressure at the base of a 35km thick crust is about 10 GPa, and realistic pressures for most naturally deformed crustal rocks range from several hundred bars upwards.

The hydrostatic stress (the experimental confining pressure) causes elastic volume changes which depend on the compressibility of the material. The size of these volume changes is unimportant except at great depth.

A more important aspect of increasing confining pressure is to cause a significant increase in both the yield stress and the failure stress, giving the material a higher effective strength whether in compression or in extension. In other words, increasing the confining pressure allows a greater amount of strain to accumulate before failure; it increases the rock ability to flow ductilely. To give a simplified explanation, moderate lithostatic pressures near the Earth’s surface allow the atoms of stressed rocks to move freely and easily break their bonds; as a result, near-surface rocks under sufficient stress can undergo brittle failure. The same rock at several kilometres depth would deform ductilely instead, because the greater lithostatic pressure impedes breakage of chemical bonds.



Shear plane orientations in function of confining pressure in experiments on marble
from: Paterson M.S. (1958) *Bull. Geol. Soc. Am.* **69(4)** 465-476

The dihedral angle between conjugate fractures widens with increasing confining pressure. The experimental observation is consistent with the Mohr criterion of failure.

Effect of pore-fluid pressure and impurities

Since the fluid pressure is hydrostatic and opposed to the lithostatic pressure, it is not surprising to summarise increasing pore pressure to be analogous to decreasing confining pressure. It reduces both the rock strength and its ductility. The combined reduced ductility and strength promotes flow under high pore pressure.
The presence of a fluid phase in rocks undergoing deformation is important in two ways.
- Firstly it can promote mineralogical reactions, particularly at elevated temperatures, which affect the mechanical properties of the rock.
- Secondly, it can reduce the effect of the lithostatic or confining pressure (the effective normal stresses) by countering the direct pressure between adjoining grains by the power of the pore-fluid pressure.

The chemical effect is often small compared to the mechanical influence of the pore-fluid pressure, which is expressed by:

$$P_{eff} = P_{lith} - P_{fluid} \tag{14}$$

where P_{eff} is the effective pressure on the solid material, P_{lith} is the confining pressure, and P_{fluid} the fluid pressure.

A **coefficient of fluid pressure**, λ is often used to denote the ratio of fluid pressure and lithostatic load)

$$\lambda = P_{fluid} / P_{lith} \tag{15}$$

Exercise

Calculate the coefficient of fluid pressure in the upper part of the crust, where fractures reach the surface. Bulk rock density is 2400 kg.m^{-3}.
Water density = 1000kg.m^{-3}, builds columns in open fractures, λ = 0.42

For saturated rocks, in which the pore-fluid pressure may be very high, the effect of the confining pressure is cancelled out and the rock strength is reduced to near-surface conditions. In boreholes, high P_{fluid} may cause fracturing of the casing or wall rocks, whereas low P_{fluid} are responsible for blow-out and borehole closure.

The effect of high fluid pressure on rocks at elevated temperature is illustrated by the stress-strain curves for wet and dry quartz crystals. The yield stress at 950°C in wet quartz is only about one-tenth of that required for the dry quartz at the same temperature. The ductility in this case is increased by the presence of water, which explains why certain materials, normally strong even at high temperature, can flow under metamorphic conditions in the presence of aqueous fluids.
By comparison, the activation energy in equations (10) and (12) is dependent on any fluid phase or impurity, for example such as depending on the diffusion of CO_2 through the calcite structure, or amounts of (OH) in the quartz structure. Impurities usually lower the yield stress values and increase the field of viscosity in the stress-strain curves.

Time factor

Time plays a prominent role in deformation. Rapid loading of **rate-sensitive** materials exhibit different stress-strain curves than those obtained under slow loading. Experimentally, the effect of a slow strain rate is analogous to increasing temperature: Decreasing strain rate decreases the rock strength and increases ductility. Values of the yield and ultimate strengths of rocks are much higher if measured over short time periods than over geologically significant time periods. Ductile flow has been observed to take place under a constant long-term load whose value is considerably lower than the elastic and plastic limits (e.g. bending of graves and candles under their own weight). Accordingly, rocks under geological load flow much below the yield strength measured under experimental conditions of temperature and confining pressure comparable to those in the earth.

Strength of materials - Interrelationship of stress, strain and time

One important variable is the **strength** of a material, which is the stress at which failure occurs. Many materials possess both a **yield strength** defined as the stress above which permanent deformation occurs, and a **failure strength** at which fracturing occurs. Laboratory measurements provide limiting values of lithospheric stress, provided that one effective principal stress is known.
The relationship between stress and strain for real materials that exhibit a combination of elastic, viscous and plastic properties depends critically on the length of time for which the differential stress is applied. In laboratory experiments with duration of up to a few days, the behaviour of the material is effectively "instantaneous" in geological terms and differs significantly from that of the same material under stresses with more geologically realistic duration of months or years. The long-term strain behaviour of materials is called **creep**. The important characteristic of creep behaviour is that values of strength measured over short time periods are much larger than those measured (or extrapolated) for long periods of time. The long-term or **creep strength** of most rocks is only in the range 20 60 % of their instantaneous strength.
Strain rate is expressed as a change in dimension per unit time, in the form 10^{n} sec^{-1}. Extrapolation of the experimental data to average geological strain rates of approximately 10^{-14} sec^{-1} indicates that rocks should have strengths of a few 100 bars at 200-300°C and may be a less than 100 bars at 800-900°C.
Also, all the relevant brittle and ductile processes are thermally activated, i.e. the rates of deformation at fixed differential stress increase with increasing temperature and time necessary to achieve a given deformation increases as temperature decreases.

STRENGTH PROFILES

Knowing the amounts and rates of displacements involved in plate tectonics, the rheology of the lithosphere has been assumed to be viscous. However, several types of observations show that the

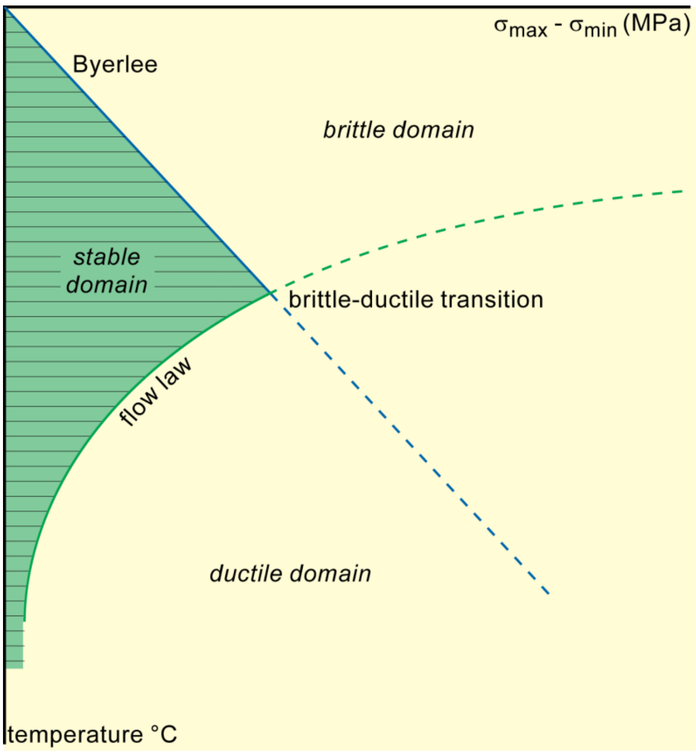
rheology changes with depth. Geologists observed that rocks from shallow crustal levels have deformed in a brittle fashion, while rocks from lower crustal levels deformed in a ductile manner. Supportively seismicity occurs within the upper crust in both strong and thin oceanic lithospheres and comparatively weaker and thicker continental lithospheres. Rheological models of the lithosphere should account for these variations. They are graphically summarized as strength versus depth profiles called **yield-strength envelopes**.
Strength profiles use intuitive relationships on the constitutive relationships that govern the rheology of rocks. The first intuitive principle is that of minimum effort: a rock will break if it requires less stress than to flow and vice-versa, according to physical conditions considered. At least two contrasting and superposed lithospheric regions are distinguished: (1) pressure-controlled regions of elastic-rigid behaviour where it is easier to obey laws for the fracture strength, and (2) temperature-controlled parts with ductile (viscous or plastic), non-linear flow laws for the yield strength.

Construction

The construction of strength profiles is based on considerations of two experimentalists whose names have been attached to the resulting model: “Brace-Goetze lithosphere”. These models combine the brittle and ductile deformation laws into a simple strength profile of the lithosphere, i.e. the profile of maximum rock strength with depth.
Assuming validity when extrapolated to macroscopic scales and geological times, the rheological data obtained from laboratory tests on the mechanics of cold and hot rocks can be assembled to produce a simple, first-order model of the mechanical behaviour of the lithosphere.

Brittle levels

The upper part of the lithosphere can be considered elastic on the time scale of hundreds of millions of years. This interpretation explains the fact that loads such as seamounts are supported by the lithosphere for long periods of geological times. Also on shorter time scales, the Earth behaves elastically as in the case of seismic events and seismic wave propagation. The main importance of the elastic behaviour of rocks for geodynamic processes lies in the fact that the lithosphere is strong and capable of supporting and transmitting large loads for long periods of times. Elasticity controls the stress field in rigid domains, which in turn controls the position and timing of deformation in non-rigid domains. This means that it is fundamentally impossible to assess the stress-state of the lithosphere without considering the effects of elasticity.
Stresses in the Earth cannot exceed the strength of rocks. The upper part of the model uses Byerlee's friction law for the limiting strength and assumes a hydrostatic pore-pressure gradient. Indeed, it is assumed that the upper crust is crossed by many discontinuities of every shape and size. The relevant deformation process is therefore sliding along a pre-existing surface. An Andersonian stress state and fault orientation (which means that one of the principal stresses is vertical and that conjugate faults make an acute angle bisected by the maximum principal stress) is also assumed.
According to Byerlee's law, the brittle strength of rocks increases linearly with depth and is independent of material. Thus, the brittle shear strength of the lithosphere will increase roughly linearly throughout the crust and mantle part of the lithosphere.



Elementary construction of a rheological profile

Ductile levels

The lower part of the model is based on the extrapolation to relevant temperatures of the steady-state power-law (equation 10). It is generally agreed that the bulk viscous behaviour of the lithosphere may be described by a non-linear, temperature and stress dependent power-law rheology. The curved strength envelope of rocks in ductile deformation is largely insensitive to pressure variations (hence to depth) but decreases exponentially with depth, due to thermal softening. Therefore, the viscous shear strength is likely to decrease with depth. Depending on the state of stress, grain size and composition, rocks with a non-linear viscosity at moderate temperature/depth may follow a linear, Newtonian constitutive law at a high temperature.

To calculate the strength of the lithosphere, a strain rate and a geothermal gradient must be applied to appropriate rocks (e.g. wet quartzite in the crust, olivine in the mantle).

Effect of stress regime

The Mohr construction demonstrates that the normal stress σ_N and the shear stress σ_S are related to the maximum and minimum principal stresses:

$$\sigma_N = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta$$

and (16)

$$\sigma_S = \frac{1}{2}\sin 2\theta\left(\sigma_1 - \sigma_3\right)$$

with θ the angle between σ_1 and the normal to the fault plane, which is the same angle as between the fault plane and σ_3 (script on stresses).

The application of the Byerlee's law to the lithosphere requires that it is reformulated in terms of the principal stresses, instead of using normal and shear stress. In effect, we need to know the total differential stress $\left(\sigma_1 - \sigma_3\right)$ sufficient to activate fault slip. For this purpose, one substitutes equations (16) into equations (8) and (9).

For $\sigma_N \le 200\,\text{MPa}$: $\quad (1/2)\sin 2\theta(\sigma_1 - \sigma_3) = 0.85\left(\sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta\right)$

For $200 \le \sigma_N \le 2000\,\text{MPa}$ $\quad (1/2)\sin 2\theta(\sigma_1 - \sigma_3) = 50 + 0.6\left(\sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta\right)$

Remembering the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$:

For $\sigma_N \le 200\,\text{MPa}$: $\quad (\sin\theta\cos\theta)(\sigma_1 - \sigma_3) = 0.85\left(\sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta\right)$

For $200 \le \sigma_N \le 2000\,\text{MPa}$ $\quad (\cos\theta\sin\theta)(\sigma_1 - \sigma_3) = 50 + 0.6\left(\sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta\right)$

Developing these equations to group σ_1 and σ_3 one obtains:

For $\sigma_N \le 200\,\text{MPa}$: $\quad \sigma_1 = \dfrac{\sin\theta(\cos\theta + 0.85\sin\theta)}{\cos\theta(\sin\theta - 0.85\cos\theta)}\sigma_3 = \tan\theta\dfrac{(\cos\theta + 0.85\sin\theta)}{(\sin\theta - 0.85\cos\theta)}\sigma_3$

For $200 \le \sigma_N \le 2000\,\text{MPa}$ $\quad \sigma_1 = \dfrac{\sigma_3 \sin\theta(\cos\theta + 0.6\sin\theta)}{\cos\theta(\sin\theta - 0.6\cos\theta)} + \dfrac{50}{\cos\theta(\sin\theta - 0.6\cos\theta)}$

Taking $\theta = \pi/3$ as standard angle for numerical application, Byerlee's law in terms of the principal stresses become:

$\sigma_1 = 4.85\sigma_3 \approx 5\sigma_3$	for	$\sigma_3 < 101\,\text{MPa}$
$\sigma_1 = 3.12\,\sigma_3 + 176$	for	$\sigma_3 > 101\,\text{MPa}$

Exercise

Draw a Mohr diagram that refers to the last set of equations, and discuss how it verifies the Byerlee's Law.

In an Andersonian stress state and fault orientation, the condition for failure is given by the maximum value of :

$$(\sigma_v - \sigma_h)$$

where σ_h and σ_v are the vertical and horizontal stresses, respectively. In the Earth, v is usually due to the weight of the overburden (equation 12). In Extension $\sigma_v = \sigma_1$ and in compression $\sigma_v = \sigma_3$. Under low stress, in extension:

$$\sigma_3 = \sigma_h \approx \sigma_1/5 = \rho.g.z/5$$

The condition for failure in extension is therefore given by:

$$(\sigma_v - \sigma_h) = 4\rho.g.z/5$$

Substituting g = 9.81 m.s^{-2} and ρ = 2800 kg.m^{-3} gives a slope of 21.97 MPa.km^{-1} for extension. For high stresses the slope is 18.6 MPa.km^{-1} + 67.7 MPa.

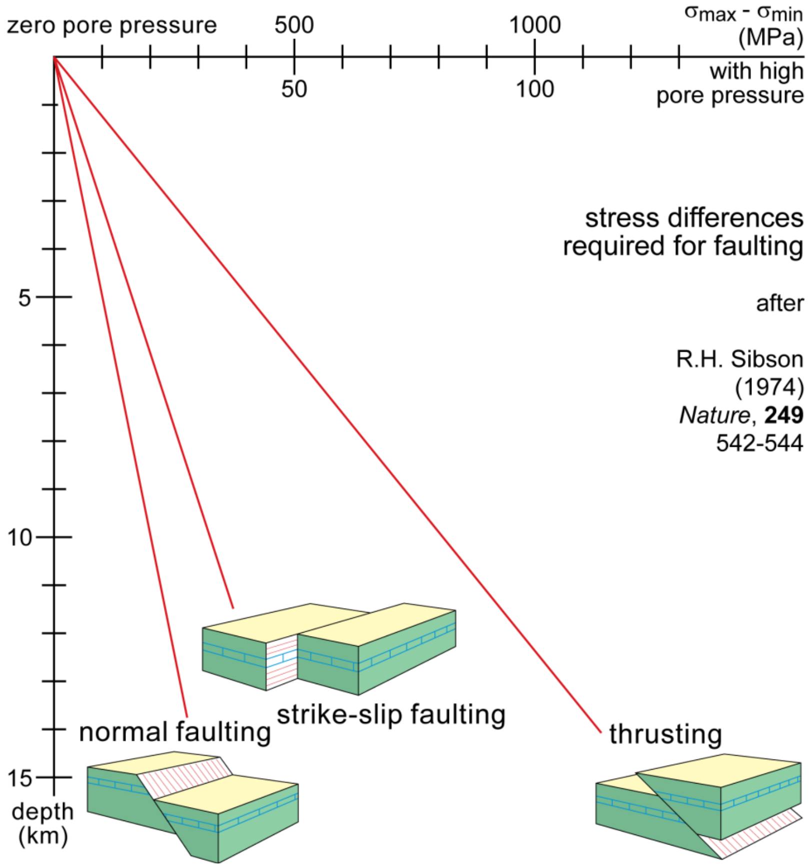
In compression failure occurs when the horizontal compressive stress has built up to a sufficient value that it has effectively balanced the horizontal stress. This occurs when:

$$\sigma_h = 5\rho.g.z \qquad (17)$$

the condition of failure is therefore given by:

$$(\sigma_v - \sigma_h) = -4\rho.g.z \qquad (18)$$

Substituting g and ρ gives a slope of -109.87 MPa.km^{-1} for compression. For high stresses the slope is –57.7 MPa.km^{-1} - 210 MPa.



The Byerlee's law tends to give a lower bound on the yield stress because it assumes that the lithosphere is already fractured. Seismic waves propagate through the lithosphere, thus suggesting that rocks are dominated by elastic behaviour and that the background stress state of the lithosphere is lithostatic. The condition of failure should be considered for a perfectly elastic material. In a perfectly elastic body the principal stresses are given by:

$$\sigma_2 = \sigma_3 = \sigma_1 \left[\nu / (1 - \nu) \right]$$

where ν is the Poisson's ratio (Equation 2), about 0.25 for rocks. Therefore, in the elastic lithosphere:

$$\sigma_1 = 3\sigma_3$$

Substituting the weight of overburden for these principal strains we see that failure will occur if:

In extension $$(\sigma_v - \sigma_h) = \rho.g.z - (\rho.g.z/3) = 2\,\rho.g.z/3 \qquad (19)$$

In compression $$(\sigma_v - \sigma_h) = \rho.g.z - 3\,\rho.g.z = -2\,\rho.g.z \qquad (20)$$

The same parameters as above yield a slope of 18.3 MPa.km^{-1} in extension and –54.9 MPa.km^{-1} in compression.

The increase of the frictional strength with depth for normal, reverse and strike slip faults all will be different because of the difference in orientation with the vertical stress. Minimum stress levels for movement on normal faults are only a quarter of those required for thrust faulting and about half that for strike-slip faulting.

Effect of pore pressure

Pore pressure modifies the frictional failure criterion (equation 7). The brittle regime for most rocks can be adequately described by the Coulomb frictional law, which in the case of pre-existing faults of favourable orientation and negligible cohesion can be written as:

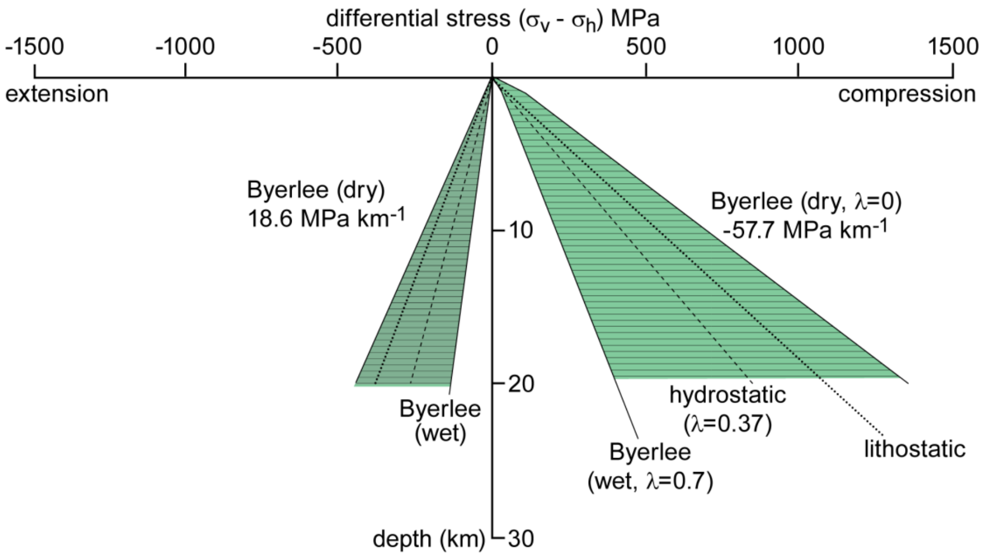
$$\sigma_1 - \sigma_3 \geq \beta \rho g z (1-\lambda) \quad (21)$$

where the same abbreviations as in previous equations represent the same terms, ρ is the average density of rocks above the depth z, and λ the ratio of pore fluid pressure to lithostatic pressure (equation 15); the typical hydrostatic value is $\lambda = 0.4$.
β is a numerical parameter depending on the type of faulting with values 3, 1.2 and 0.75 for thrust, strike-slip and normal faulting, respectively.
The governing equations are

in compressional faulting $$(\sigma_1 - \sigma_3) = \frac{2\left[c + \mu\rho g z (1-\lambda)\right]}{\left(\mu^2+1\right)^{1/2} - \mu} \quad (22)$$

in tensional faulting $$(\sigma_1 - \sigma_3) = \frac{-2\left[c + \mu\rho g z (1-\lambda)\right]}{\left(\mu^2+1\right)^{1/2} + \mu} \quad (23)$$

in strike-slip faulting $$(\sigma_1 - \sigma_3) = \frac{2\left[c + \mu\rho g z (1-\lambda)\right]}{\left(\mu^2+1\right)^{1/2}} \quad (24)$$



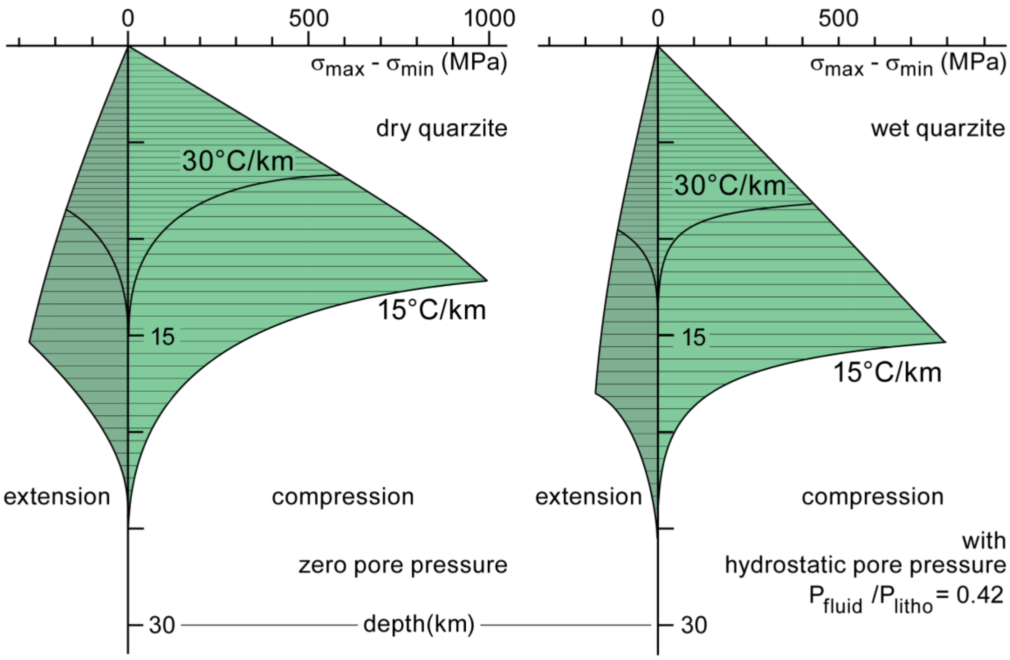
Strength profiles for dry and wet conditions in the crust

These equations (with μ the coefficient of friction) show that the stress required for slip will be lower if the confining effect of lithostatic pressure is reduced by higher fluid pressure, which increases λ.

Description

A yield strength envelope shows that the relative importance of temperature and pressure changes in the brittle and ductile regimes of rock deformation. The result is asymmetric and emphasises the strength of the brittle-ductile transition significantly bigger and shallower in compression than in extension. The area under the yield strength envelope leads to the **integrated strength**, which is a measure of the total lithospheric strength. This implies that (1) the integrated yield strength transmits the global plate tectonic stress field and (2) the driving forces of plate tectonics cannot

exceed the integrated lithospheric strength. This provides an important constraint on the geodynamics of oceans and continents.
The strength predicted in the brittle, frictional part of the crust depends only on the assumed pore pressure. As the slope of the brittle yield strength envelope depends on the mode of faulting, two profiles are plotted on both sides of the vertical temperature axis. One on the positive side of stresses refers to the profile in compression (equations 20 and 22), the one on the negative side corresponds to the profile in tension (equations 19 and 23).

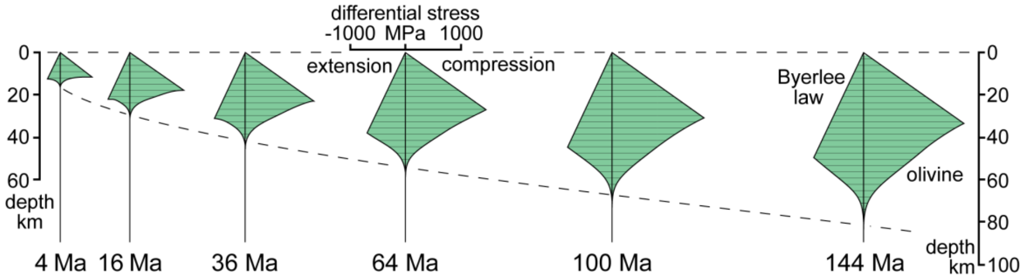


Crustal strength profiles for constant strain rate of 10^{-14} s^{-1}
Effects of pore pressure and temperature gradient

The strength predicted in the ductile, deeper part depends strongly on the assumed rock type, temperature and strain rate. Usually a quartz rheology is applied for the continental crust. Rocks change with depth and the same sort of predictions can be made for any mineral phase. Different viscous envelopes become relevant and the lithosphere may appear rheologically stratified. In particular at the Moho, mantle (olivine) rocks are more creep resistant and stronger than other silicates at the same temperature.
Byerlee's law combined with the quartz and olivine flow law provides a maximum stress profile to about 25 or 50 km, respectively. For a temperature gradient of 15°C/km, stress will be close to zero at the surface and reaches a maximum of 600 MPa (quartz) or 1100 MPa (olivine) for hydrostatic pore pressure.
The rheological stratification of oceanic lithospheres differs from that of continental lithospheres.

Oceanic lithosphere

Strength profiles of oceanic lithosphere have simple shapes. The basaltic crust has a near 0 km thickness at the mid oceanic ridge and the thin sedimentary cover is too negligible to influence the overall rheology of the whole lithosphere. The oceanic crust thickens to about 5-7 km by cooling the sub-crustal mantle lithosphere. Thus, the bulk composition of an oceanic lithosphere is rather uniform and the rheology of olivine should govern its bulk behaviour. Strength profiles first increase linearly with depth according to the Byerlee law, then decrease exponentially according to the olivine viscous power law to grade into the nearly no strength of the asthenosphere (etymology: strength-less sphere). Since Byerlee laws apply whatever the type of rocks, the presence of plagioclase and pyroxene in the cold crust of basalts and gabbros can be omitted. The brittle and viscous yield envelopes intersect at the brittle-ductile transition.



Strength profile of the oceanic lithosphere in function of age

During cooling, the depth of the brittle-ductile transition increase and the size of integrated region below the strength profile varies. In other words, the bulk strength of the oceanic lithosphere increases with age until the lithosphere reaches a steady-state thickness of about 90-100 km at the 60-80 Ma age. Whatever this age, the profiles are characterized by a single strength maximum.

Exercise

Construct the thickness evolution of the oceanic lithosphere from the MOR outward. Assume that the 5km thick oceanic crust at the MOR does not change with time. Conversely, the thickness (D) of the oceanic mantle lithosphere varies according to:

$$D(\text{km}) = 10 * \sqrt{\text{Age in Million Years}}$$

Calculate the thickness and the average density of the lithosphere when it is 2.5, 5, 10 and 20 Ma old. Use 2900 kg.m^{-3} as density of the oceanic crust and 3300 kg.m^{-3} for the mantle. At which age and thickness the lithosphere reaches the same average density as the asthenosphere (3250 kg.m^{-3})? Comment this result.

Continental lithosphere

The concept is more complex to apply to continents where the crust is compositionally heterogeneous and thicker than in oceans.

Continental crust

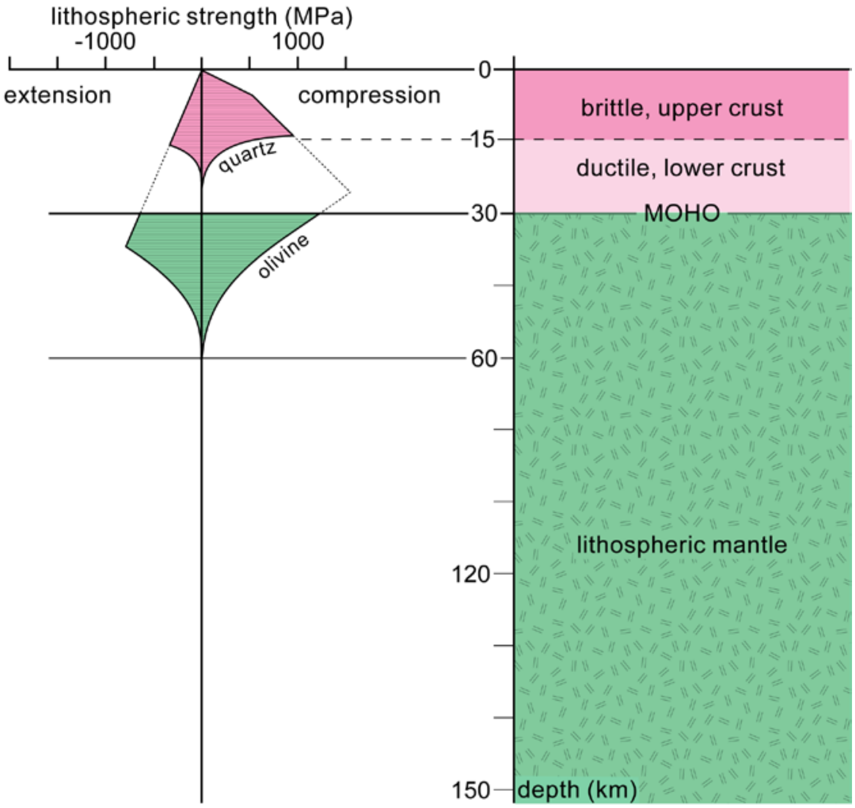
The rheology of the crust is generally approximated as that of the most common mineral, wet quartz, which is brittle at shallow depth but typically ductile at temperatures >300-350°C, well above the Moho (about 500°C). If the crust deforms at constant strain rate, then its brittle strength increases with depth and crosses the viscous strength envelope at the "brittle-ductile" transition, at about 15 km depth for a typical geothermal gradient of 20° C km^{-1}. The crust above this depth deforms in a brittle fashion. Below this depth the crust is viscous and the viscous strength decreases with depth. The brittle ductile transition supports the highest shear stresses anywhere in the crust and it is within this depth range that the highest moment release might be anticipated from intraplate earthquakes. At increased strain rate, the viscous stresses will become larger so that the brittle ductile transition moves downwards and vice versa.

Mantle

Below the Moho, the viscous curve for wet quartz is replaced by that for olivine. Olivine supports substantially higher shear stresses than quartz and the Moho is therefore the region of highest strength in the lithosphere. As for the oceanic lithosphere, the thickness of the continental mantle lithosphere is age-dependent, being the thickest under oldest cratons.

Total lithosphere

Known experimental data seem to indicate that the lower crust is markedly weaker than the peridotite mantle immediately above the Moho. As a consequence, the continental lithosphere has one (at the bottom of the crust) or two (the previous one plus another at mid-crustal levels) soft ductile layers sandwiched between brittle layers.



idealised strength profile of a continental lithosphere

The prevailing model (commonly referred to as the “jelly sandwich” model) is that of a strong upper crust, down to about 15 km, overlying a soft middle to lower crust (down to 20-30 km) and a strong lower crust and uppermost mantle down to the depth at which the olivine viscous law applies. On a rheological profile, the strong brittle levels reaching the Byerlee line are superposed and alternate with strength indentations corresponding to the various ductile levels. The continental lithosphere is thus rheologically layered. Owing to their shape, these diagrams are popularly called “Christmas trees”. A rheologically stratified lithosphere has resulted in generally good agreement with the depth distribution of earthquakes in oceanic and continental lithospheres.

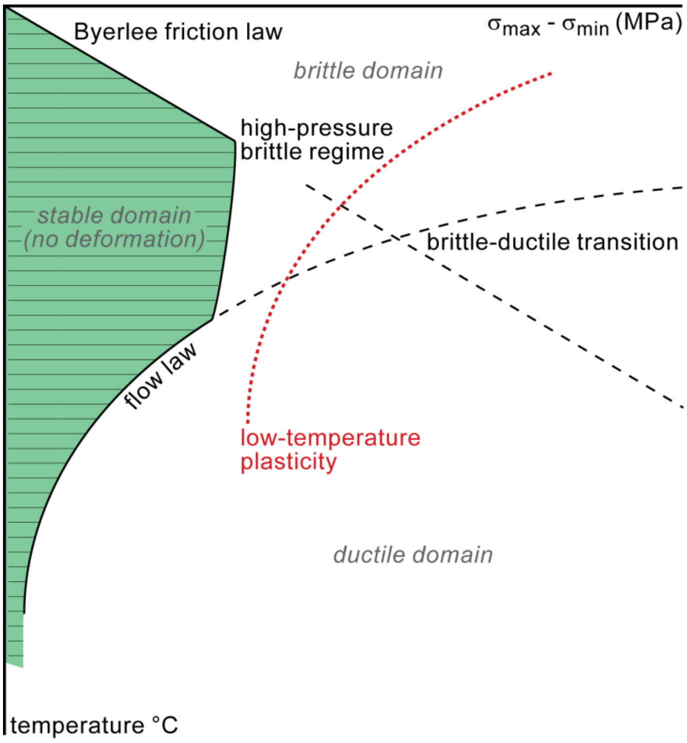
In this model, a significant part of the total strength resides in the lithospheric mantle. Where the geothermal gradient is very high, the thickness of the viscous levels increases which causes both lower crust and mantle to be softer. Following the same reasoning, the rigid levels will be much thicker in old, cold cratonic areas than in areas of thinned lithosphere with high heat flow, or in thickened orogenic zones. This may account for a long-termed focussing of lithospheric deformation into orogenic zones. Once a part of the continental lithosphere has become a locus of internal deformation and crustal thickening, it may concentrate the deformation for a long time, until it cools below the plastic limit of crustal rocks or the tectonic forces cease.

The rheological layering may be complicated if the lower crust is mafic or granulitic. In that case the rheology of plagioclase or dry quartz, respectively, is applied. Molten rocks may be significantly weaker, resulting in decoupling horizons where contrasting rheologies occur. In that

perspective, the continental mantle can be attenuated by positive thermal anomalies due to, for instance, plume heads.
In that manner, one may include many layers and quite a variety of strength profiles for the continental lithosphere. Yet, the integrated strength for the continent will usually be less than that of the ocean of same age.

Strength of the brittle-ductile transition

Different geotherms, strain rates and thicknesses are used to generate rheological models covering a variety of geodynamic settings. On any model, at some depth, the flow law curve intersects the line representing Byerlee's law.



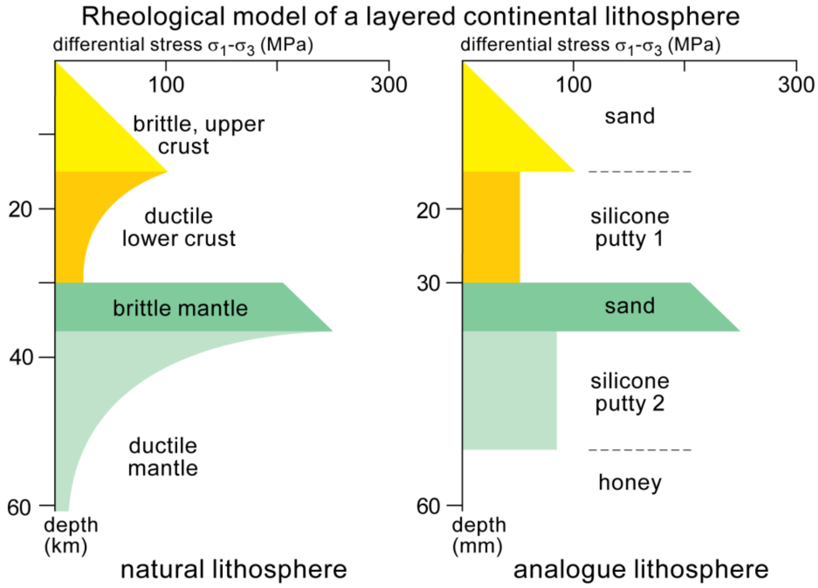
Unscaled effects of high pressure brittle regime and low-temperature plasticity on strength profiles

The intersection of the brittle and ductile laws is taken to mark the brittle-ductile transition in the Earth. It is the strongest part of the strength profile.
Friction laws are experimentally verified only for confining pressures up to a few hundred MPa, corresponding to the upper crust. In profiles, it is routinely extrapolated to lower crustal and even upper mantle depths. This extrapolation is valid only if there is no difference between the type of shear fracture at low and high pressure. Experimental evidence for granite suggests that a change in fracture mechanism takes place within the brittle field with increasing pressure. The fracture strength at high confining pressure has weaker pressure dependence than frictional fracture strength, and decreases with increasing temperature. The brittle zone appears divided into two parts: The upper frictional one where strength increases linearly with depth and a lower one where strength is approximately constant with depth. The result is to lower the strength of the brittle-ductile intersection with some sort of cut-off.
Other mechanisms such as low-temperature plasticity may limit the deviatoric stresses that can be sustained. No experimental data are available for this plasticity field. Low-temperature plasticity can be expressed by a non-linear constitutive law similar in shape as equation (11), but shifted

mostly towards lower temperature for similar deviatoric stresses. The result is that the brittle-ductile intersection occurs at lower temperatures and stresses as on the simple plot.

Application

The rheological knowledge is fundamental to laboratory, scaled experiments on analogue models. These tectonic models are attempts, in miniature, to simulate conceptually and mechanically the deformation of the Earth's lithosphere (in particular large-scale features such as sedimentary basin formation or orogen development) under defined loading conditions. Analogue models supplement numerical modelling because the inherent limitations of each method are different.



In theory, the lithospheric rheological profiles give a lower limit of the strength of the lithosphere, because they describe the stress levels required to drive suitably oriented faults. In practice, they only provide a qualitative guide to the actual strength, i.e. the greatest possible stress as a function of depth. These profiles show that most of the total strength of the lithosphere resides in the seismogenic crust. It is clear that the thermal structure of deforming plates, and therefore their age, is very important to the mechanics of the phenomenon. Such models are used to analyse observed variations in the crustal style of deformed continental and oceanic lithospheres.

Limitations

First order uncertainties limit a direct assessment of lithosphere rheology. An exact knowledge of the composition of rocks belonging to the lithosphere under consideration is impossible. However, rocks are heterogeneous mineral aggregates constituted of a few main components.

Composition

The first assumption is that the main constituents govern the rheology of aggregates. The composition of continental lithosphere is envisioned from three information sources: (a) seismic data (b) petrology of surface exposures, and (c) xenolith data. All lead to very broad rheological approximations.

The upper continental crust composition is usually dominated by granitoids, quartz-rich sediments, and quartz-bearing schists. Quartz is therefore considered to control the bulk rheology of the upper continental crust. Since the upper part of the crust is hydrated, the creep parameters of wet quartzite are taken as best approximation.

The oceanic crust is basaltic and wet. Wet basalts are therefore the reference rheology.

The bulk of the continental lower crust generally has seismic velocities compatible with mafic composition. However, more felsic compositions may predominate in particular regions. Accordingly, the mechanical behaviour of the continental lower crust is modelled with the creep parameters of both mafic and felsic granulites, and also of wet and dry diabase.
This crustal stratification rests on an ultramafic upper mantle. Thus the flow properties of olivine and peridotites should be representative of the response of the upper mantle. A wet rheology may be appropriate for continental lithospheric mantle in zones recently affected by subduction of oceanic lithosphere and post-Paleozoic tectonothermal events, while a dry rheology may be more relevant for older regions.

State variables

A second type of uncertainty stems from errors in the state variables as pressure, temperature, stress level, grain size and fluid pressure. Temperature and grain size assert a direct control on the creep of rocks, whereas the effective pressure controls the frictional strength of fault.
Thermal conditions in the continental lithosphere are modelled using assumptions on the depth distribution of heat producing elements and either a fixed asthenosphere temperature or heat flux boundary conditions at the lithosphere / asthenosphere boundary. Thermal cooling of the lithosphere introduces an age-dependence in the rheology of the lithosphere.
The lithostatic pressure can be estimated as the overburden load with reasonable boundaries. The effective pressure (equation 14) is much less certain. However, typical crustal permeability is such that fluid pressures in excess of lithostatic cannot be maintained for geological times.
The shear stress level ranges from 1 to 150 MPa at depths between 1 and 100 km. Linear extrapolation of the Byerlee’s law probably overpredicts the stresses at depth greater than 5-10 km (fitting stresses in the KTB borehole at 8 km, but only 150 MPa at 11 km in the Kola well).

Conclusion

The strength of the lithosphere (i.e. the total force per unit width necessary to deform a lithospheric section at a given strain rate) is a function of composition, crustal thickness, and geotherm. Confining pressure increases rock strength.
Increasing temperature weakens rocks.
At depth, temperature overcomes the strengthening effect of confining pressure, which generally leads to ductile behaviour, by opposition to frictional processes that dominate at low temperatures.
The lithosphere tends to have horizontally stratified mechanical properties. Different factors contribute to this stratification. Generally, pressure, temperature and composition are strongly dependent on depth and vary relatively less laterally. The horizontally stratified compositions reflect layering in undeformed sediments or stacks of structural units, such as thrust sheets.
Lithospheric strength profiles illustrate, approximately, which portions of the crust behave in brittle and ductile manners. The first-order rheological behaviour (brittle or ductile) at any given depth is determined by the relative magnitude of frictional and creep strength. However, the actual variation of strength with depth depends strongly on the deformation mechanism as well as on the type of rock. The brittle / ductile transition is likely to be gradual, but this is a second order effect that is often neglected.

The large-scale characteristics of a lithospheric system show a satisfactory agreement with the inferred rheological structure: sub-horizontal decrements should be the rule rather than the exception where the lithospheric rheology is strongly stratified. The most significant factor, are the maximum in strength at the brittle-ductile transition in the continental crust and a second maximum at the Moho, where the rock composition changes from quartzo-feldspathic rocks to olivine-dominated peridotite. Comparison of the high temperature bulk steady-state rheologies of crustal rocks with flow laws for polycrystalline olivine indicates that the Moho should be a rheological discontinuity with the uppermost mantle being significantly stronger than the lowermost continental crust.

Rocks are generally weaker at lower strain rate. The strain rate has a decisive influence on the rheology of rocks: materials deforming under low strain rates (geologically realistic) exhibit creep, whereas high strain rates are associated with geologically instantaneous strain.
Fluids weaken rocks by affecting bonding of materials while high fluid pressure reduces effective stress.

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