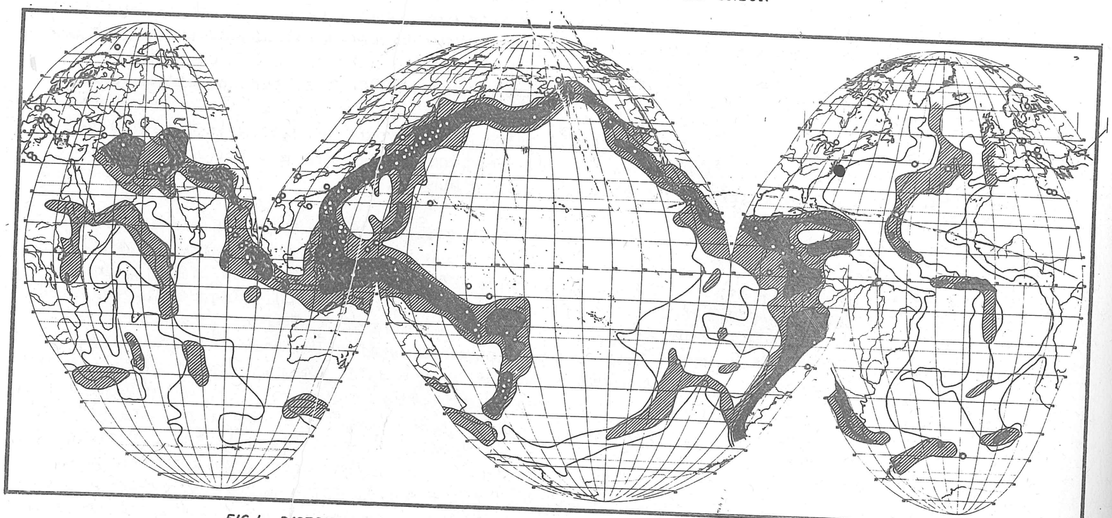

FIG. 1 — DISTRIBUTION OF EARTHQUAKES APRIL 1, 1935, TO MARCH 31, 1936
SMALL CIRCLES INDICATE EPICENTERS

total number of deaths in earthquakes during the past two centuries averages 28,000 per year. The average annual rates for certain important earthquake-districts, he gives as follows: 13,862 in China; 7980 in Colombia and Ecuador; 2240 in Japan; 1006 in Italy; 991 in Central America; 715 in India; 586 in Persia; 550 in Asia Minor; 99 in Greece; 35 in the Philippines; 29 in Formosa, and 6 in the United States. A few districts, such as Spain, Portugal, Venezuela, and Algeria, are not accounted for but, grouping them together, the corresponding number of lives lost per year is 380. The sum of all the districts is 30,479. This is an enlightening summary and may be taken as one of the reasons for the continued interest in the earthquake-problem.

As will be seen from Figure 1 the distribution of the earthquakes during 1935-36 is quite normal, possibly one or two earthquakes falling out of recognized earthquake-zones. In the United States itself we consider California as furnishing our most important laboratory for earthquake-study. However, during the calendar year 1935 there were ten earthquakes which caused some damage. Eight of these occurred outside of California, two in Montana, two in Arizona, one in New Mexico, one in Nebraska, one in Georgia, one in Canada -- the last shook the entire northeastern section of the United States, causing some damage in New York State.

There are three outstanding earthquakes for this past year. The first occurred April 21, 1935, in northwestern Formosa. The second occurred May 31, 1935, at Quetta, India, and the third occurred October 18, 1935, in Montana. The Canadian earthquake of November 1, 1935, was felt over an area of probably 650,000 square miles, but only slight damage resulted except in the area of the immediate epicenter which was very sparsely populated. In the Formosa earthquake 3410 people were killed, 12,000 were wounded, 17,900 houses were destroyed, and 21,400 houses partly collapsed or were severely damaged.

The houses of this section are of two types, namely, the usual wooden houses of Japan and those of brick and mud-blocks cemented with mud-mortar with roofs of thatched straw or Formosa tile. The latter are excellent protection from heat but can not withstand violent earthquakes. The ratio of the percentage of wooden houses destroyed against the mud-brick type was as one is to six in spite of deterioration of wooden houses by the work of the white ant. From evidence on the surface it has been determined that the relative horizontal movements were as great as 1.4 meters and the vertical movements ranged from 0.5 to 2.5 meters. From the estimated value of acceleration (less than one-third gravity) T. Suzuki, J. Astron. Geophys. v. 13, no. 1, pp. 55-59, 1935) says that this earthquake was not as strong as the Tango earthquake of March 1927 or the North Idu earthquake of November 1930.

Reporting on the Quetta earthquake, W. West, Assistant Superintendent of the Geological Survey of India, gives the following information (Records of the Geological Survey of India, v. 69, part 2, pp. 203-240, 1935): "From the point of view of the number of lives lost the Quetta earthquake is considered the most disastrous within historic times. Like all Baluchistan earthquakes this was of shallow focus, about five miles and, therefore, not felt over a large area (100,000 square miles) as compared with nearly 2,000,000 square miles for the North Bihar earthquake of 1934. About four shocks were felt in the epicentral zone in the preceding ten weeks and about ten seconds preceding the main shock a slight tremor was felt by a sentry, Quetta being a strongly fortified region. Sixty hours after the main shock a severe aftershock occurred which, had there been any houses left to destroy, would have completed the destruction." Although this earthquake occurred in the early morning hours it was observed, nevertheless, by those participating in military maneuvers at the time of the earthquake and about four miles north of Quetta. The ground-motion was described as like "a terrier shaking a rat." "The ground heaved as in a raging sea or as the way a small boat behaves in the wake of a large storm. Those who did not lie down at once were either flung down or were just able to stagger about."

The towns of Quetta, Mastung, and Kalat were almost entirely razed. Twenty-six thousand out of 40,000 population were killed. Including those who were killed in the surrounding villages, the total loss of life runs up to 40,000 or more. This enormous loss of life was due largely to poor construction and narrow streets. Some new governmental buildings collapsed, and examination proved failure was due to an inferior quality of mortar. Destruction was greater near the junction of rock and alluvium than in the central area of alluvium. The author states this was probably due to the greater disturbance caused by the different periods of vibration of the two types of material. Damage was also greater on wet alluvium than on dry, water aiding in the transmission of the shock. Wire-netting in plaster caused many people to become trapped and made rescue difficult and self-help impossible. An interesting illustration is the effect on two bungalows located across the street from one another, one designed as earthquake resistant and the other not. No damage occurred to the former, while the latter was completely destroyed. Mr. West recommends buildings of one story, square in shape. Some buildings had galvanized-iron chimneys above the roof-levels. Such chimneys stood while the brick-chimneys fell.

The Montana earthquakes of October 18 and 31, 1935, killed six persons and did property damage to the amount of $5,500,000. Dr. D. S. Carder of the United States Coast and Geodetic Survey was an eye-witness of the second shock. Part of his report is quoted herewith:

"On the morning of October 31, I investigated some solid waves which were caused by the earthquake of October 18 with Ray Hofstatter of the Montana Power and Light Company. At 11:37 a.m., Mountain Standard Time, we had just returned to the Federal Building and as we were discussing the previous earthquake while sitting in Mr. Hofstatter's parked car, which was facing north, a sharp blow came from the northeast and deep underground and a second later the car began a violent rocking from side to side. The rocking was immediately preceded by a second hard blow. At the time I was thinking of the fine record we were getting and of a brick-wall hidden by an awning just across the sidewalk. The latter, however, was quite safe. Otherwise I was not frightened but was highly excited. After the heavy shaking subsided, I took my leave to tend the instruments.

"The ground continued to tremble for several minutes and strong aftershocks were frequent. Business activity was demoralized and it was reported that several thousand people, especially women and children, evacuated the city. The unrest of the ground seemed to continue throughout the day and into the night with repeated low rumblings, many of them not accompanied by perceptible shocks.

"The amplitude of the ground in the frequency of the greatest agitation was probably no more than one or two mm, yet the upper floors of some of the buildings must have swayed an inch or more; some observers say a foot or more though this may be exaggerated. All observers agree that poles and trees swayed violently. Some observers claim they saw waves allegedly a foot high and 50 or 100 feet from crest to crest in the streets.

"The earthquake of October 31 caused considerable new damage to buildings apparently untouched by the former hard tremors, although it is possible that many of these had already been weakened. Earlier damage was greatly accentuated. Several buildings which had been badly hit by the former tremor were completely demolished on the 31st and many others were rendered uninhabitable."

As in the Long Beach earthquake the schools suffered severe damage and the time of occurrence was again fortunate in that these buildings were unoccupied. The new $300,000 building, normally occupied by 1000 students, is practically a total loss. Had the times of occurrence of these two quakes been reversed, loss of life and suffering would have been enormous. It is as though Nature has sounded her warning for the people to look to the safety of their school buildings and other public gathering places. We can not hope to always be as fortunate in the time of occurrence of earthquakes as we were in those of Long Beach and Helena.

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REFLECTION DIP-SHOOTING METHODS IN SEISMIC PROSPECTING

H. M. Rutherford

Most writers on reflection-methods in seismic prospecting base their interpretations on a calibrated velocity. This is to say, they assume for the purposes of computation an isotropic and homogeneous overburden. This conception has proved fruitful in practice because it reduces the computations to a minimum. This does not imply, however, that the interpretations can only be made by use of an average velocity.

When the reflection-method was first put into use the object was to map some particular marker. For instance, depths would be computed on a limestone, for example, the Viola Lime in Oklahoma, and these plotted on a map. Contours would be drawn and thus the presence of a structure might be detected. This method would not work, however, in the Gulf Coast Region of Texas, for there were no beds which could be correlated by this method over a wide region. This gave rise to the so-called "dip-method" of reflection-shooting. In this method the seismograms were interpreted for reflections and from these data the dip of the bed which caused the reflections was computed. No depth-calculations were essential. The basic principles of this method have been given by the present author in a previous paper (H. M. Rutherford, Reflection-methods in seismic prospecting, Geophysical Prospecting, Amer. Inst. Mechanical Engineers, 1934).

Consider the situation as depicted in Figure 1. In this Figure O is taken as the origin of a disturbance, such as that caused by a dynamite-blast. P_1, P_2, and so on are the points at which energy is reflected from the interface shown. The distances from O at which the respective reflections are recorded are X_1, X_2, and so on. I is the image-point of the reflections. The interface slopes to the surface at the angle w. θ is the angle between the normal to the interface and the surface. It is easily seen from the Figure that

$$V^2t^2 = X^2 + 4H^2 - 4HX \cos \theta$$

or

$$V^2t^2 = X^2 + 4H^2 + 4HX \sin w$$

where t is the time corresponding to the distance X, taken in the direction of increasing depth of the interface. In case we shoot "up-slope," that is, the distances increase with decreasing depth of the interface, we have

$$V^2t^2 = X^2 + 4H^2 - 4HX \sin w$$

This can be put in the form of a hyperbola by multiplying $4H^2$ by $(\sin^2 w + \cos^2 w)$, and factoring, or

$$t^2/(4H^2\cos^2 w/V^2) - (X \pm 2H \sin w)^2/(4H^2 \cos^2 w) = 1$$

where the plus sign corresponds to shooting "down-slope" and the minus sign means shooting "up-slope." From this it is seen immediately that we get the minimum time and distance when we shoot "up-slope" and the time and distance will be given by

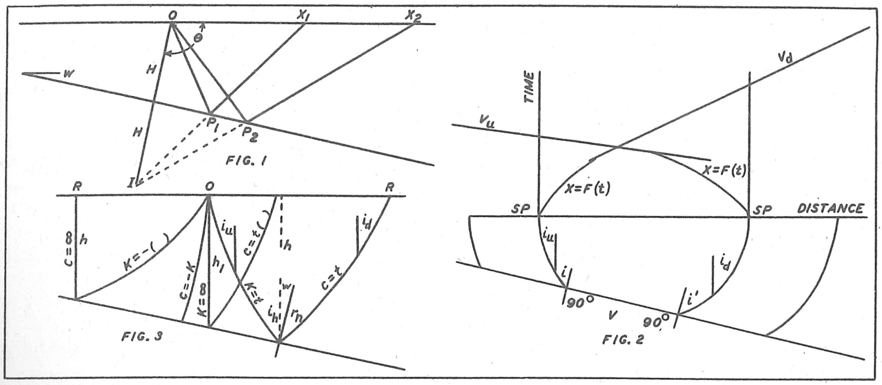
$$X = 2H \sin w$$
$$t = 2H \cos w/V$$

Thus as we shoot "up-slope" from the shot-point the time of reflection will decrease with increasing distance up to the distance which is equal to 2H sin w. Beyond this point the time will increase with increasing distance.

It will be noticed that the slope can be told qualitatively immediately from the record. If the bed slopes up from the shot-point, the reflection-times will be less and less with increasing distances, provided the distance does not exceed 2H sin w in amount. If down, the reflection-times will be greater with increasing distances. Curves can be graphed from the foregoing equations or approximate formulas can be devised for field-use which will give the slope.

In actual practice profiles are shot across country. If there occurs a reversal in dip, then an anomaly is present. By taking enough strikes and dips the structure can be drawn in.

The author has presented elsewhere a method for mapping structure in the case of horizontal beds by assuming an overburden which has a velocity which varies with the depth below the surface (The interpretation of reflection-seismograms, Trans. Amer. Geophys. Union, June 1933). In



another paper he gives an actual example where the velocity-depth function is determined and used to compute a depth (A formula for weathering correction, Trans. Soc. Pet. Geophysicists, v. 5, March, 1935). It is the purpose here to develop a method whereby the slope and the depth can be determined from reflected phases which are caused by a discontinuity which slopes to the surface and which has an overburden in which the velocity varies as the depth below the surface.

We will assume, in what follows, that we have already determined the velocity-depth function in each case. An actual example of the determination of this function will be given. If we consider a refraction-profile shot "down-slope" it is easy to show that the incident ray will make an angle with the normal such that

$$\sin i_u = V(y)/V_u$$

If we shoot in the opposite direction we have

$$\sin i_d = V(y)/V_d$$

where V_u is the velocity obtained by shooting "up-slope" and V_d is the velocity obtained by shooting "down-slope." These velocities are "apparent." Figure 2 shows the situation. These rays will travel such a path that the angles in question are the "critical angles."

Suppose that instead of V_u and V_d we write K and C, that is,

$$\sin i_u = V(y)/K$$

$$\sin i_d = V(y)/C$$

The above means that the ray-path will strike a hypothetical bed at the critical angle whose velocity shooting up is K and whose velocity shooting down is C.

The above considerations suggest a way to obtain all the possible reflected paths from a discontinuity. If we take the depth of the discontinuity below the surface as h, we have then, if

$K = V_u$ and $C = V_d$ $\qquad$ $i_h, r_h = i_c$ (critical angle)

$K < V_u$ $\qquad$ $i_h, r_h > i_c$

$K > V_u$ $\qquad$ $i_h, r_h < i_c$

Thus if we consider all possible values of K, and hence of C, we will describe all possible reflection-paths.

In Figure 3 consider a ray-path which originates at O and after reflection at the point P returns to the surface and is recorded at the point R. Call the depth of P below the surface, h. Then the time to travel this path is given by

$$T_r = t_{OP} + t_{PR}$$

or

$$T_r = \int_0^h K\, dy/(V(y)\sqrt{K^2 - V^2(y)}) + \int_0^h C\, dy/(V(y)\sqrt{C^2 - V^2(y)})$$

The distance corresponding to this time is

$$X = \int_0^h V(y)dy/\sqrt{K^2 - V^2(y)} + \int_0^h V(y)dy/\sqrt{C^2 - V^2(y)}$$

Note in Figure 3 that at depth h, where i, r are the angles of incidence and reflection, respectively, at this depth, i = r

$$i = i_u + w$$

$$r = i_d - w$$

$$i_d = i_u + 2w$$

We have

$$\sin i_d = \sin (i_u + 2w) = \sin i_u \cos 2w + \sin 2w \cos i_u$$

Since

$$\sin i_d = V(h)/C$$

$$\sin i_u = V(h)/K$$

we obtain by substitution

$$V(h)/C = V(h)/K \cos 2w + \sqrt{K^2 - V^2(h)}/K \sin 2w$$

hence

$$C = (K\, V(h))/(V(h) \cos 2w + \sqrt{K^2 - V^2(h)} \sin 2w)$$

In the case where w = 0, C = K and

$$T_r = 2\int_0^h K\, dy/(V(y)\sqrt{K^2 - V^2(y)})$$

$$X_r = 2\int_0^h V(y)dy/\sqrt{K^2 - V^2(y)}$$

We will now consider in some detail the equation which gives the relation between the two parameters C and K and the angle of slope, w. If K be taken as infinite (positive), then the ray will leave the shot-point orthogonal to the surface, that is, $i_u = 0$. In this case we have

$$C = V(h)/\sin 2w$$

and as a consequence $i_h = r_h = w$. Note that i_u is taken positive measured from the vertical to the right and that i_d is measured positive to the left of the vertical. If C be taken to be infinite it will be seen that we are shooting "up-slope" and

$$K = V(h)/\sin(-2w)$$

If we are shooting "up-slope" we have

$$C = K\, V(h)/(V(h) \cos 2w - \sqrt{K^2 - V^2(h)} \sin 2w)$$

If C and K are taken equal but opposite in sign, then we have a ray-path which is reflected from the discontinuity such that it strikes it orthogonally. This will happen when we take

$$K = V(h)/\sin w$$

Note that since the bed is sloping the proper value of h must be substituted in the time- and distance-integrals. If we take the distance O to P to be h_1 when K is infinite, we have

$$h = h_1 + \left[\int_0^h V(y)dy/\sqrt{K^2 - V^2(y)}\right] \tan w \quad \text{"down-slope"}$$

$$h = h_1 - [\int_0^h V(h)\, dy/\sqrt{K^2 - V^2(y)}]\tan w \quad \text{"up-slope"}$$

If only small distances be taken on the surface, that is to say K is large, then the last term can be neglected. Or, if w is sufficiently small the second term can be neglected.

We consider a simple example. Suppose that the velocity-depth function is such that V(y) = V, a constant. Let the angle of slope, w = 10° and the depth OP be 5000 feet. We have then

$$t = \int_0^h C dy/(V\sqrt{C^2 - V^2}) + \int_0^h K dy/(V\sqrt{K^2 - V^2})$$

$$x = \int_0^h V\, dy/\sqrt{K^2 - V^2} + \int_0^h V dy/\sqrt{C^2 - V^2}$$

$$h = h_1 + [\int_0^h V dy/\sqrt{K^2 - V^2}]\tan w$$

From which we obtain, after integrating

$$t = hK/(V\sqrt{K^2 - V^2}) + hC/(V\sqrt{C^2 - V^2})$$

$$x = Vh/\sqrt{K^2 - V^2} + Vh/\sqrt{C^2 - V^2}$$

$$h = h_1/(1 \pm V/\sqrt{K^2 - V^2}\tan w)$$

In the last expression, the plus refers to "up-slope" and the minus to "down-slope." We take h_1 = 5000 feet, w = 10°, and V = 10,000 feet/second. If we choose K = +100 then C will be 23.026905. We will be shooting "down-slope" and the distance will be 2966 feet and the time 1.0767 seconds. With K = -100 then C = 40.594861 and we will be shooting "up-slope." The time and distance are X = 755 feet and T = 1.00068 seconds. If we take K = -57.5871, that is to say

$$K = -V/\sin w = -10/\sin 10° = -57.5871$$

(In all the calculations kilo-feet was used instead of feet.) The value of C will be for this value 57.5871. This assures us that the reflection will take place orthogonal to the reflecting surface. The time and distance will be X = 0 and t = 0.98481 second. Consider now the minimum time and distance. This will be such that C = infinity. Thus the value of K will be K = -V/sin 2w = -29.238056 and C = very large (infinity for all practical purposes). The time and distance will then be X = 1.710 kilo-feet and t = 0.96985 second. All of these figures can be obtained, of course, from the formula as given before

$$V^2t^2 = X^2 + 4H^2 \pm 4HX\sin w$$

The method as outlined would not be the best way to calculate the simple case just cited. Its value lies in the more complicated problems connected with a velocity-depth function which cannot be represented by a constant. Many examples might be given but we restrict ourselves to just one on account of space.

The writer has found several areas in which the velocity-depth function could be closely approximated by $V = ay^{1/2} + b$. In a region in Oklahoma the time-distance relationships could be expressed by $X = 0.711519\, t^2 + 5.172t$ where the constants are given in kilo-feet. By the well-known formula for penetration versus velocity it was found that

V in feet/sec	Depth in feet	V in feet/sec	Depth in feet	V in feet/sec	Depth in feet
6000	341	6600	785	7200	1350
6200	474	6800	961	7400	1563
6400	623	7000	1149	7600	1787

A function of the form $V = ay^{1/2} + b$ fits these data thus

$$V = 2.130239\, y^{1/2} + 4.7292 \qquad \text{RMSE} = 16 \text{ feet/sec}$$

All measurements are in kilo-feet.

For a function of this form the time-distance relations are

$$T_r = \int_0^h \frac{K\,dy}{(ay^{1/2} + b)\sqrt{K^2 - (ay^{1/2} + b)^2}} + \int_0^h \frac{C\,dy}{(ay^{1/2} + b)\sqrt{C^2 - (ay^{1/2} + b)^2}}$$

$$X_r = \int_0^h \frac{(ay^{1/2} + b)dy}{\sqrt{K^2 - (ay^{1/2} + b)^2}} + \int_0^h \frac{(ay^{1/2} + b)dy}{\sqrt{C^2 - (ay^{1/2} + b)^2}}$$

$$h = h_1 \pm \int_0^h \frac{(ay^{1/2} + b)dy}{\sqrt{K^2 - (ay^{1/2} + b)^2}} \tan w$$

The reader can perform these integrations quite easily by making the substitutions $u = ay^{1/2} + b$ and $dy = 2(u-b)du/a^2$. The resulting formulae are somewhat long and will not be written here. In spite of the apparent length of calculations the formulae can be used to a good advantage to plot graphs. These graphs can be used in the field to determine the dip of the reflecting horizon.

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TRANSFORMING THE STEREOGRAPHIC MAP

F. W. Sohon

The stereographic projection, because of the simplicity and elegance of its mathematical properties, has been a mathematician's playground ever since the time of Hipparchus. In endeavoring to explain some of its properties a few years ago, I chanced to hit upon several that certainly do not seem to be well known. One of these "The rule for longitude" was announced in a paper read by Father Buckley at our Fordham meeting two years ago. Another property, concerned with transforming the projection, forms the subject of the present paper. It is well known that the stereographic projection can be transformed into either an orthographic, or gnomonic, or equal-area, or equal-distance map, merely by displacing the points radially. These are transformations of the stereographic map into other maps possessing other properties. In this paper, however, we are interested in another sort of transformation. We must first digress to explain the concept of inverse in a circle.

Inversion in a circle is a problem in plane geometry. In a plane, one point is said to be the inverse of another point with respect to a given circle (or straight line) if every possible circle (or straight line) drawn through the first point orthogonal to the given circle (or given straight line) also passes through the second point. When two points are inverse with respect to a given straight line (see Fig. 1), they are symmetrical with respect to that line. If two points are inverse with respect to a circle (see Fig. 2), they lie upon the same radius of the circle, but their position is such that the product of their distances from the center of the circle is equal to the square of the radius of the given circle. We can also explain the matter in a different way. If we draw the diameter of the given circle which passes through the point whose inverse is required, and place on the opposite end of the diameter a concave mirror whose principal focus lies at the center of the circle, then the image of the given point in the concave mirror will be the inverse of the given point with respect to the given circle.

With a circle given, the inverse of any point can thus be found readily enough by measuring its distance from the center of the circle, dividing the square of the radius of the circle by the quantity just obtained, and laying off the quotient along the radius in the direction of the given point. The inverse can also be found geometrically as follows: In Figure 3, let C be the center of the given circle and P be the given point. Draw any other arbitrary radius of the circle CQ, then from the point R, where line CP cuts the given circle, draw RS parallel to PQ intercepting CQ in S. Lay off CT on CP equal to CS, then the point T will be the inverse of the point P with respect to the given circle, as is evident from the similar triangles, CRS and CPQ. If we refer to Figure 4 instead of Figure 3, the construction will be found for a point that is given inside the given circle. The same identical construction applies to Figure 4 without any change whatsoever. Of course, if the point is given on the circumference of the given circle then it coincides with its own inverse.

Let us now consider two points that are given on the surface of a sphere. If one of these points is the image of another point with respect to a plane of symmetry through the center of

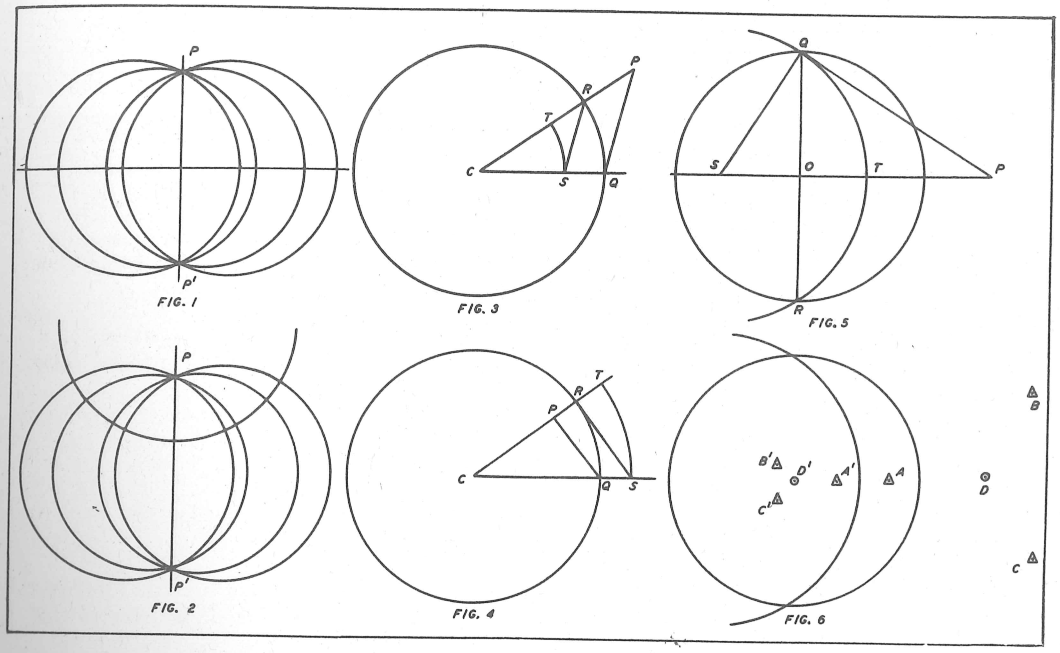

FIG. 1 FIG. 3 FIG. 5
FIG. 2 FIG. 4 FIG. 6

the sphere, then every circle on the surface of the sphere which passes through both will have its pole on the great circle in which the plane of symmetry cuts the sphere; and consequently, all circles through both points cut the great circle in question orthogonally. Now the stereographic projection is conformal. Circles are projected as circles or as straight lines, and angles are projected true. Hence, if two points on the surface of the sphere are symmetrical with respect to a given plane, their projections will be mutually inverse with respect to the circle (or straight line), which is the projection of that great circle which the plane of symmetry traces on the surface of the sphere.

This principle can be used to change the zenith of the map. Referring to Figure 5, let it be required to transform the stereographic map so that the point whose projection is P will be the zenith of the new map. Let O be the center of the primitive circle, then P and O must be mutually inverse with respect to a circle which is the projection of a great circle. Draw the line OP and the diameter QOR of the primitive circle perpendicular to OP. Draw QS perpendicular to QP, intersecting OP in the point S. Then since $\overline{SQ}^2$ is equal to SO multiplied by SP, the points O and P will be inverse with respect to a circle QTR, whose center is S and whose radius is SQ. This circle will be the projection of a great circle because it passes through the two points, Q and R, which are situated at the extremities of the diameter of the sphere. If we now invert all of the points of the map in the circle QTR, we shall have the projection of a configuration of points on the surface of the sphere that is symmetrical to the original configuration, and the point whose projection was originally the point P has now become the zenith of the map. If we refer to Figure 6, the construction has been carried out so as to make D' the inverse of the point D fall at the center of the primitive circle. A', B', and C' were then found as the inverse of A, B, and C, respectively. It will be observed that the new map has the defect that it is obtained by reflection in a plane, instead of by a rotation, and that consequently it is drawn in reverse. This can be remedied by taking its mirror-image, or by holding the paper to the light and looking at the new map from the back.

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THE DETERMINATION OF GROUND MOTION FROM SEISMOGRAMS

A. Blake

(Abstract)

The greatest source of error in determining the motion of the ground from a seismogram is the inaccuracy which enters in determining the position of the center of oscillation of the seismometer-pendulum as a function of the time, that is, the error of recording and scaling. This error becomes large in the strong-motion problem as the period of the pendulum recedes in either direction from the range of interest; this conclusion is not dependent on the assumption that the motion is of simple harmonic type.

Denoting by x_1 the displacement of the seismometer, by x_2 the corresponding quantity for a damped normal component of a building subjected to the ground motion, and by x the displacement of the ground, we may write the seismometer equations in the form

$$(D - r_1)(D - r_1')x_1 = -D^2x = (D - r_2)(D - r_2')x_2$$

Solving for x_2 in terms of x_1 we find

$$x_2(t) = x_1(t) + m_1 \int_0^t e^{-m_2(t-s)} \sin m_3(t - m_4 - s)\, x_1(s)\, ds$$

in which m_1, m_2, m_3, and m_4 are functions of the constants of seismograph and building, and are all real. (This form is dependent on the inequality of r_2 and r_2', but many of the results for $r_2 = r_2'$ may be obtained as limits from this equation. In the cases $r_1 = r_1' = 0$ and $r_2 \doteq r_2' \doteq 0$ we have the solution of the seismometer-equation and an approximate solution of the inverse problem. The solution of the latter problem may be obtained from the seismometer-equation by double integration, but the present method is also useful.)

The same equation holds for the error δx_1 and the resulting error δx_2 in the building motion. The upper bound of $|\delta x_2|$ for the most unfavorable buildings and earthquakes is our measure of the error. Denoting by b an upper bound for $|\delta x_1|$, we find by inspection an error-function which maximizes $|\delta x_2|$; it is a function which is everywhere equal to +b or -b and whose sign agrees with that of $\sin m_3(t - m_4 - s)$ except at time t, when it agrees with that of m_1, or the negative of such a function. We expand this function in Fourier series; the integral may be evaluated term by term although the Fourier series does not converge uniformly. In the unfavorable case of a long-continued earthquake and a feebly damped building the first term predominates, and in the limit becomes infinite in comparison to the other terms, so that the study of this term suffices for the purpose of estimating the maximum error due to any error-function δx_1.

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SEISMOGRAPHIC TILT-MEASUREMENTS AT BUFFALO

John P. Delaney

(Abstract)

With instrumental arrangement as previously described at the Pittsburgh and Ottawa meetings, together with additional data from thermograph, barograph, and spirit-level, three distinct types of ground-tilting have been found. Tilting in one direction only has been studied, along a north-east-southwest line, because this is the only direction of noticed tilt on the Wiechert pendulum.

The first type of tilt is irregular, a few tenths of a second of arc and of duration five to 20 hours. The tilt appears related to changes in the barometer rather than in temperature, and probably it is caused by steep barometric gradients when these occur along the line of orientation of the instruments.

The second type of tilting has an annual period, amounting to about three seconds of arc southwesterly in spring and summer and northeasterly in fall and winter. This annual tilt appears deep-seated in the rock foundation of the pier, since temperature-changes, either in the vault or in outside weather, seem to cause no immediate tilting.

The third type of level-change appears to be a constant southwesterly tilt affecting both the Wiechert and Wood-Anderson seismometers. Displacement of the Wood-Anderson zero-position between January-February 1935 and the same months in 1936 indicates a tilt-accumulation in the course of the year of approximately 0.7 second of arc. Further spirit-level data will be required for confirmation and measurement of this tilt, and for study of its persistence.

Canisius College,
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LOCAL EARTHQUAKES IN NEW ENGLAND, 1934-1935

Mary P. Collins

(Abstract)

During 1934 and 1935, records of approximately 150 disturbances, definitely local in character, were obtained on the Benioff seismographs at the Harvard Seismograph-Station. Distances for 105 of these shocks were computed from the S-P interval, using the Harvard travel-times for New England. The range of distances varies from 12 to 320 km, with a total of 60 shocks between 100 and 150 km from the station.

An attempt to establish criteria for distinguishing blast-records from those of local earthquakes is discussed. It is concluded that most of the records investigated represent natural earthquakes, in spite of a curious concentration in daytime hours.

Seismograph-Station,
Harvard University,
Cambridge, Massachusetts

THE NEW SEISMIC VAULT AT FORDHAM

J. Joseph Lynch

(Abstract)

The William K. Spain Seismic Station of Fordham University was moved from its former position on the campus to the site it now occupies, adjacent to the Physics Building, Freeman Hall. A diagram was shown to demonstrate the set-up of the piers, six in number, with the center pier, a semicircular one, enabling the adjustments of instruments from either side with greater ease. The new waterproofing cement was described, as well as the dehumidifying unit which has been giving excellent results since its installation.

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CORRELATIONS BETWEEN TILTING OF THE GROUND AND THE TIDES IN CHESAPEAKE BAY

George Merritt

The interferometric tiltmeter, described in greater detail in previous issues of the Transactions, consists of a shallow basin containing a liquid, and a flat glass cover-plate on which is engraved the reference-marks. The surfaces from which the interfering reflections are obtained are the upper surface of the liquid, which remains level, and the lower surface of the cover plate, which tilts with its support. These surfaces are set about 4 mm apart. For visual observation a simple combined illuminating and viewing instrument is used but for purposes of study it seemed desirable that a recording device be attached.

The recorder (Fig. 1) was developed and tested under a grant of the National Research Council. Several experimental machines were made and the most successful one produced so far has been sent to the University of California where a tiltmeter-station is maintained in cooperation with the Seismological Section of the United States Coast and Geodetic Survey.

The recording apparatus consists essentially of a camera, substituted for the eyepiece of

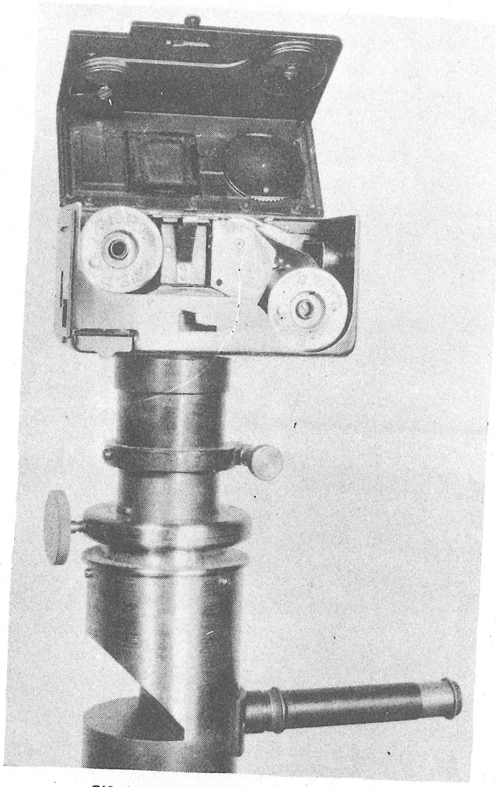

FIG. 1—AUTOMATIC TILTMETER-RECORDER

the illuminating and viewing instrument. This camera uses 32-mm supersensitive panchromatic perforated moving-picture films. With these films an exposure of 10 to 15 seconds has been found sufficient to obtain good pictures of the fringe-system formed by the yellow emission of helium. One week's record (hourly pictures of the fringes) consists of a strip of film 10 to 12 feet long, which is easily filed and provides a permanent record for future study. The film may be examined by projecting it, one frame at a time, on the wall at a magnification of about 30X or 40X.

Power for winding the film and operating the shutter is provided by a weight suspended from a cable wound on a drum. The timing mechanism is a double-spring power-drive clock operating a cam which makes and breaks an electric circuit. The electrical power is furnished by a six-volt storage-battery, its function being to light the helium tube used as a light source and to operate a solenoid which releases a trigger, allowing the suspended weight to operate. A yellow filter is introduced which suppresses the other emissions of the helium lamp so that the yellow fringes are clear and sharp. A side-entry telescope is provided to enable the operator to inspect the fringes without disturbing the camera or opening the shutter.

While operating the recorder at Cove Point Landing on the Chesapeake Bay, a tide-gage was installed and maintained by the United States Coast and Geodetic Survey for a period of two months. This was situated in seven feet of water, opposite to the tiltmeter, about 50 yards from shore and about 200 yards from the tiltmeter, which is on the first high ground.

A correlation between tide-load and tilt was found to be small but positive. The simple formula $r = \sum XY/n\ S_x\ S_y$ was used. In this formula $S_x = \sqrt{\sum X^2/n}$, $S_y = \sqrt{\sum Y^2/n}$, X and Y are the deviations from the mean of the tilt and tide, respectively, and n is the number of pairs used. The value $r = 0.17 \pm 0.06$ was found, using 514 pairs of observations.

Since barometric pressure, temperature, and ground-water probably also introduced tilts it is not to be expected that a very large correlation would be found. The softness of the strata (the tiltmeter is set in blue clay) may also lessen the amount of observable effect. On the other hand, the situation is favorable in that the tide phase is such that the loading effect is not greatly masked by the tide-producing forces.

A record of the barometric pressure was kept by means of a barograph furnished by the United States Weather Bureau but so far no correlation has been found between any component of the tilt and the pressure.

U. S. Coast and Geodetic Survey,
Washington, D. C.

A DISCUSSION OF SOME PROBLEMS IN EPICENTER WORK

R. R. Bodle

During the past year a number of seismological questions have presented themselves. I will discuss three of them very briefly here. The first has to do with the problem of regional variation in the travel-time of seismic waves. An unusual example of this situation has been found, namely, the shock of November 30, 1935, in the Caribbean Sea near Panama (see Fig. 1). The epicenter in this case was adopted as 10°.1 north latitude and 79°.5 west longitude The distance from this point to each of the recording stations was then computed, using a formula for great-circle distances without correction for the figure of the Earth. These distances were then used to take out the (P-O) values from three different sets of well-known seismological tables. The (P-O) values were then subtracted from the arrival-time of the first wave at each station. The results in seconds may be seen in Figure 1. Each station is here indicated by an approximate direction-line from the epicenter. At the outer end of each direction-line is found the name of the recording station and the mean value of origin in seconds as determined from the three sets of tables. Note that these mean times for the group of stations from Buffalo to Harvard are about four seconds later than any of the other mean times of origin. This indicates that the travel-time of the preliminary earthquake-waves to the stations in this sector was slower by four seconds.

This one example suggested the making of further examinations of a similar nature. Because of the difficulties frequently encountered in making locations of the shocks along the west coast of Mexico, it seemed desirable to treat some of them with this method of analysis. So far it has been possible to examine two of these shocks.

Figure 2 shows the result for the shock of May 8, 1933, in the State of Guererro. The data were not plentiful but were of a positive nature with first arrivals as indicated by the letter e for emergent and the letter i for impulsive type. Note that the time of origin at station J is in the same range as the origin-times for stations A, B, and C. The range in times of origin at the remaining stations is much larger--37 to 46 seconds. Under the high standard of operating

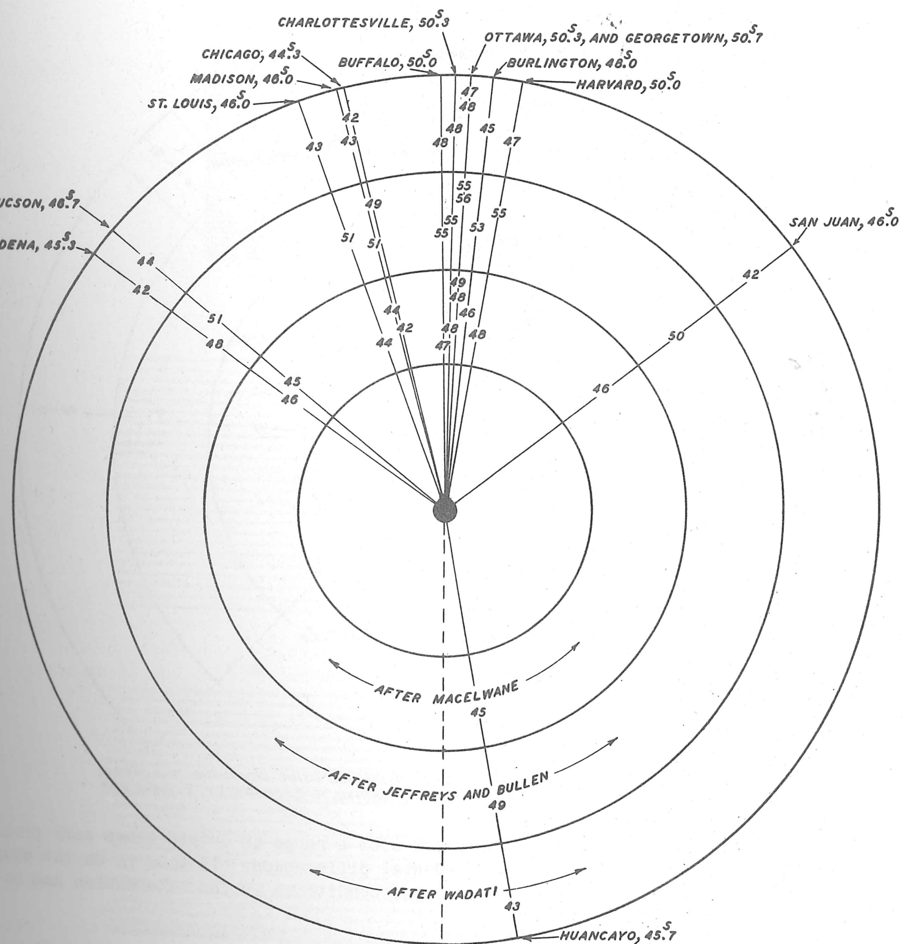

FIG. 1—SHOCK OF NOVEMBER 30, 1935; EPICENTER 10.1 N, 79.5 W; MEAN ORIGIN-TIME, $O = 3^h 39^m 48^s$
DIAGRAM SHOWING (a) STATION, (b) STATION-DIRECTIONS, AND (c) SECONDS OF ORIGIN-TIMES BY (P-O)-VALUES AFTER TABLES OF MACELWANE, JEFFREYS AND BULLEN, AND WADATI

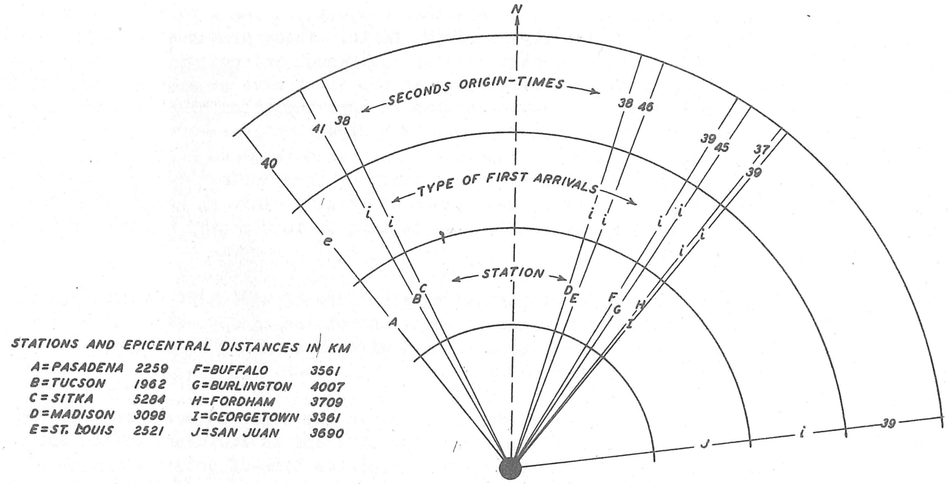

FIG. 2—MAY 8 1933
EPICENTER 17.0° N, 101.0° W; O=10h 33m 42.2s

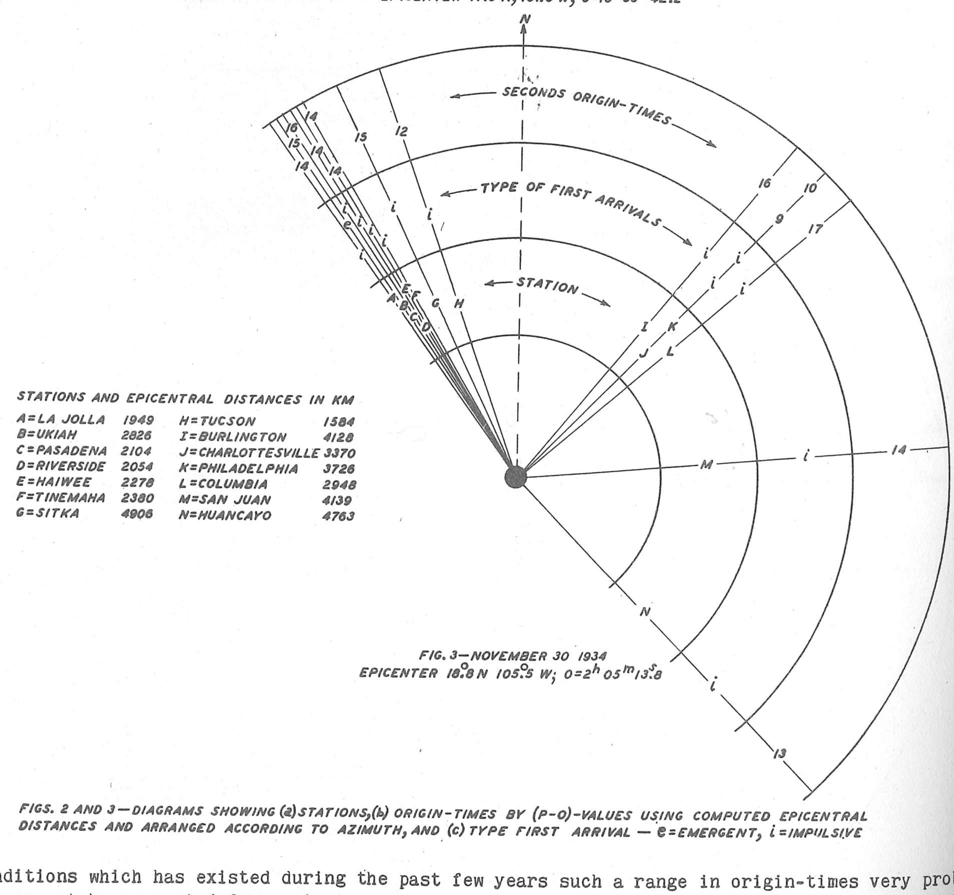

FIG. 3—NOVEMBER 30 1934
EPICENTER 18.8° N 105.5° W; O=2h 05m 13.8s

FIGS. 2 AND 3—DIAGRAMS SHOWING (a) STATIONS, (b) ORIGIN-TIMES BY (P-O)-VALUES USING COMPUTED EPICENTRAL DISTANCES AND ARRANGED ACCORDING TO AZIMUTH, AND (c) TYPE FIRST ARRIVAL — e=EMERGENT, i=IMPULSIVE

conditions which has existed during the past few years such a range in origin-times very probably can not be accounted for as instrumental or accidental differences. If this is so the only explanation left is that these origin-times picture real conditions in the retardation and acceleration of the travel-times of seismic waves.

A like examination was made for the shock of November 30, 1934, in the region of Colima, Mexico. Note from Figure 3 that the range in origin-times for the stations to the northeast is similar to the ranges shown in Figure 2. Were it not for the definite indication of regional variation shown in Figure 1 one would be inclined to overlook the situation shown in Figures 2

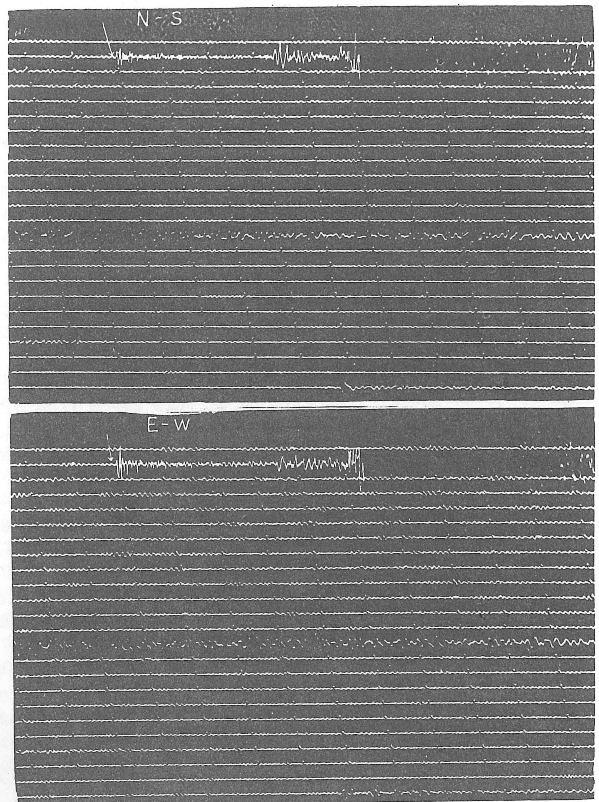

FIG. 4—SEISMOGRAM, SITKA, ALASKA, OCTOBER 31, 1935

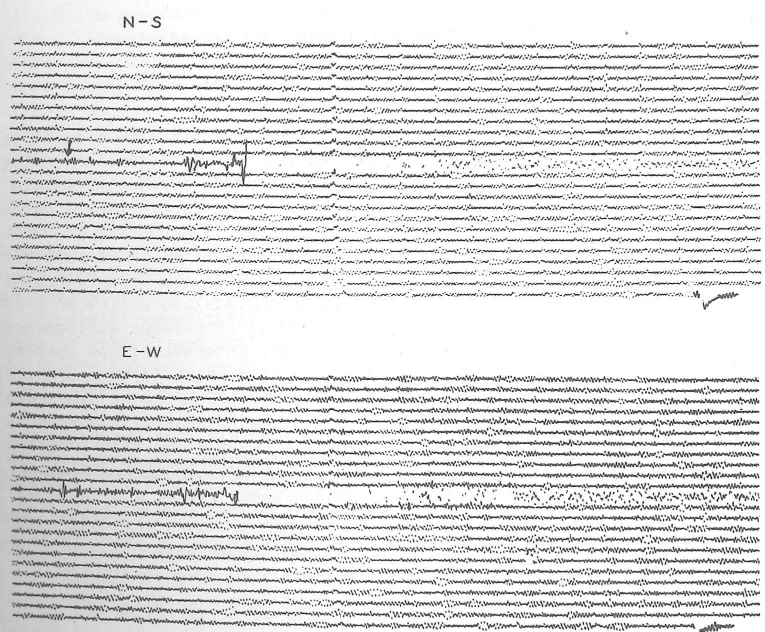

FIG. 5—SEISMOGRAM, SITKA, ALASKA, OCTOBER 18, 1935

and 3. In all three cases, however, there are rather definite indications that the transmission of seismic waves from the Mexico-Central America Region to stations in the eastern half of the United States is irregular in nature. Future studies may possibly bring forth the reason.

The second problem considered has to do with the Attica, New York, shock of August 12, 1929. Was it deep focus in nature? It seemed possible that this might be so in view of the 200-kilometer depth assigned to the Temiskiming shock of November 1, 1935. After computing the distances to the various stations reporting, the data were plotted and compared with the travel-time charts for several depths. There was no indication that this shock was other than normal in nature.

The last point has to do with the recording of the Montana shocks of October 18 and 31, 1935, at Sitka, Alaska.

Figure 4 shows the record at Sitka for the shock of October 31, 1935. The amplitude of the P-phase on both components is to be noted. The azimuth of this shock from Sitka was about 118° from north. The amplitudes of the P-phases indicated an azimuth approximately 130° from north--a quite reasonable agreement considering the possible effect of the microseisms on such a determination. Note the character of the microseisms.

In Figure 5 the amplitude of the microseisms is larger and may perhaps account for the unusual situation which we find. The P-phase on the north-south component is not recorded at all unless it is evidenced by one slight irregularity in the microseisms. Is this a correct picture? If it is correct and if the recording has not been influenced by microseisms, there must be considerably more to learn about the transmission of seismic waves. So far the writer has been unable to find any indication of malperformance of the instruments. Daily damping-tests appear on the records for both days and the apparent satisfactory registration of the microseisms seems to indicate that the equipment was operating properly. In addition the decay-tests run off October 8 indicated that the equipment was in proper order. One example such as this is not sufficient to prove the case but it would seem to indicate that azimuth-determinations from a single station may be seriously impaired by the effect of microseisms.

U. S. Coast and Geodetic Survey,
Washington, D. C.

DEEP-FOCUS EARTHQUAKES FROM A GEOLOGIST'S POINT OF VIEW

W. T. Thom, Jr.

Thanks to the recent work of the seismologists, it is evident that the time is ripe for further major advances along what Father Macelwane has termed "the geologico-seismological frontier" [see 1 of "References" at end of paper], and it therefore seems appropriate that students of structural geology should so far as possible actively contribute toward current and future collaborative study -- both as to how the Earth's crustal, subcrustal, and nuclear portions are constituted, and as to the nature of the dynamic processes which characterize and control the recurrent crustal and subcrustal deformative movements. To the writer, it appears that the most immediate and important direction in which geologists can assist in seismological-geological research lies along the line of cooperative study of deep-focus earthquakes, the discovery of which by the seismologists has opened an exceedingly interesting new chapter in earth science. The writer's endeavor, consequently, will be both to mention certain time, space, and dynamic relationships which may multiply and qualify the multiple working of hypotheses now being used in the study of the origin and tectonic significance of deep-focus earthquakes, and to mention certain matters of current experience which may suggest how and why deep-focus earthquakes occur. Because much more information obviously must be had before anyone can demonstrate that he has <u>the</u> correct answers as to how and why earth deformation takes place, a large proportion of the paragraphs to follow will consist merely of summary statements of personal opinion, the details and derivation of which will be given only as and if the views expressed prove to be sufficiently interesting to bring forth inquiry or discussion.

When it first became fairly well established that major earthquakes can and do originate at depths of several hundred kilometers, the questions at once arose as to how such postulated occurrences could be reconciled with current ideas as to how isostatic readjustments of crustal balance take place [2]; and as to the relative rôles being played by tangential thrust and (vertical) isostatic movement in those regions featured both by recent active mountain-building, and by present-day earthquakes of great focal depth. Furthermore, there naturally became manifest a tendency in some quarters to assume that "isostasy was disproved" by the indicated origination of earthquakes at great focal depths, and it is on this point that an opinion will first be expressed.

Even omitting all geodetic evidence, the geologic information in hand tends very convincingly to support the idea that isostatic readjustments have occurred, and are still occurring in some manner -- doubtless quickly and at a definite depth -- in regions underlain by thin, blister-like, bodies of molten rock; somewhat less quickly, and by a distributed movement (flow by re-crystallization), in regions underlain by highly heated, but unmelted rocks; and slowly and spasmodically by multiple shears (as in a dry glacier) in regions underlain by relatively cold sub-crustal rocks [3]. Combining the geodetic and geologic evidence, it seems particularly clear that the general concepts as to the existence and reestablishment of some form of isostatic equilibrium of the Earth's crust cannot yet be lightly discarded by students of earth mechanics.

Assuming that some form of "isostasy" <u>is</u> to be taken as a cardinal working hypothesis, what relations do the vertical motions incident to isostatic adjustment bear to the tangential thrusts which predominate in the original production of mountain-systems? The evidence indicates that isostatic readjustment tends to take place very slowly and more or less continuously, whereas mountain-building movements, under tangential thrust, seem to occur relatively much more suddenly, and in a series of major rhythms or cycles, modified by rhythmic subcycles of at least second and third orders of magnitude. Present evidence, also, indicates that the primary, linear, mountain-systems produced by tangential thrust are commonly more or less modified and multiplied by secondary mountain-folding, resulting from "landslide" movements in geosynclinal areas. During these "landslides" great sedimentary sheets or prisms become tilted beyond their angles of repose, due to deepening of geosynclines, and glide off toward the geosynclinal axis with consequent folding, crumpling, and thrusting -- as has demonstrably occurred in the Bearpaw-Highwood Region of Montana [4, 5], and as has apparently occurred in the Alps and elsewhere.

In summary then, so well as one may judge from present evidence, the geologic record suggests that, due to a more or less continuously and progressively developing lack of adjustment between the Earth's crustal area and volume, elastically-stored tangential compression recurrently mounts within the crustal rocks until the strength of the crust is first approached and then exceeded -- attended first by the accelerated development of geosynclines, and then by crustal rupture, with consequent thrusting and folding of the geosynclinal rocks into mountain-ranges -- events which characteristically mark the culmination of orogenic episodes. During the period of gradual stress-accumulation the Earth apparently tends (very slightly) to approach a tetrahedral form, with slight emergence of the continental shields marking the corners of the "tetrahedron"; whereas, during the shorter periods of orogenic stress-release, the Earth again tends to return relatively rapidly to a more nearly spheroidal form.

However, even granting that deformation under tangential compression is manifestly effective to depths of tens of kilometers, and even if "isostatic" compensatory movements apparently extend to considerably greater depths, the question yet remains as to whether we must not appeal to other causes than those involved in orogeny and isostasy to provide the large amounts of energy, locally released, which give rise to deep-focus earthquakes. For example, may not these energy-releases be due to chemical or physical transformations, rather than to more immediately dynamic causes [6]? Present opinion seems to be that there is little probability that all or most of the deep-focus earthquakes may be due to "explosions" consequent upon the subsidence of mineral substances to depths and pressures beyond their zone of stability; and a somewhat greater possibility that either an irregular heating and cooling may be responsible [7], or that sudden physical inversions -- attended by volume changes, possibly with gas evolution -- may occur suddenly at great depth, as it evidently did occur at shallow depth at the time of the recent Lassen Peak eruptions.

Happenings which are possibly more suggestive of how deep-focus quakes originate are, perhaps, afforded either by the gas "outbursts" occurring in coal and salt mines (which have been described in the Transactions of the American Institute of Mining Engineers); by the gas-eruptions and mud-volcanoes of various oil-field regions; or by crypto-volcanic and related structural features.

In the case of known mine "gas-outbursts", gas under heavy pressure apparently occurs adsorbed upon, or occluded in coal or salt. Residual tectonic stresses are latent in parts of the bed mined, and consequently when mining activities "trip the trigger", crushing of the bed begins with accelerating liberation of compressed gas from the large surface-area exposed by crushing, producing in turn a further accelerated crushing of more of the host-rock, so that, until the local residual stress is relieved, great quantities of gas and comminuted coal (or salt) are explosively discharged during a brief period of time, often causing serious loss of life as well as of property.

In the great mud-volcanoes of the Caspian region, it has been reported, natural gases, occurring at depths of some miles, recurrently have accumulated pressures sufficient to blow out the water and mud filling the throats of the vents, thus, at times, giving rise to quite violent, explosive eruptions. A comparable phenomenon has also been observed at Seneca Lake, New York [8], where natural gas, occurring under high pressure in a nearby gas-field, has escaped into the thick glacial deposits filling the bed of the Lake, until these deposits were repeatedly so saturated with compressed gas that the slight reduction in pressure produced by the passage of a barometric "low" over the Lake "touched off" sudden and violent gas-eruptions. Similar sudden outbursts of gases or vapors of much deeper origin evidently must also have been responsible for the production of some, if not all, of the "cryptovolcanic" structures studied in Europe and North America, and of the "outbursts" which were productive of the diamond pipes of South Africa which, it has been estimated, must have come from depths of at least 70 km, if not more. The possibility that gases, slowly accumulated and suddenly released, can be responsible for the production of at least some deep-focus earthquakes is, therefore, suggested for what it may be worth.

The more probable cause of most deep-seated earthquakes is, however, apparently to be sought in the operation of that set of tectonic conditions which the writer is accustomed to describe as characteristic of "basin mechanics". It is a matter of common knowledge among geologists that, when areas of flat-lying bedded rocks are warped into the form of a basin or trough, a gliding tends to occur on each bedding plane in the warped series above its neutral surface -- the peripheral portions of each bed within the basin tending to glide laterally toward the basin-margins over the surface of the bed next beneath -- whereas, beneath the neutral surface of the deformed series nearly vertical tension cracks develop, with a tendency toward a progressively greater gaping toward the bottom. It is equally true, though as yet apparently generally either not understood or appreciated, that, if a flat-lying layer of massive crystalline rocks ten or a hundred or a thousand kilometers thick, is similarly and rapidly warped into a basin or trough form, such warping can occur <u>only if</u> ruptures occur <u>above</u> the neutral surface of the deformed mass, permitting of similar glidings, and if actual or potential tensional cracking occurs below that neutral surface. Such vertical cracks or planes of lowered strength and pressure quite conceivably provide the channels whereby the basic intrusions and lava-flows of geosynclinal zones approach or reach the surface; and whereby the materials composing the diamond "pipes", and the Great Dike of South Africa, have risen from relatively great depths.

It seems highly significant that the deep-focus earthquakes thus far reported have not, for the most part, occurred in regions where rapid, shallow mountain-folding is progressing, but, rather, beneath nearby regions of greater crustal strength, within which rapid basin- or trough-development may be incipient. Most deep-focus earthquakes thus far known have been reported from regions bordering the Himalayan, Andean, East Indian, and Japanese orogenic belts, though an important exception (if not <u>the</u> exception) to this rule has been provided by the deep-focus quake which recently occurred near Timiskaming, Ontario, in the heart of the stable Canadian Shield area. This area, characterized both by great structural competence and by low temperature-gradients (that is, by relatively very cold -- and hence strong -- subjacent rocks) is one which is undergoing rapid readjustments in consequence of the recent disappearance of the last great ice sheet.

In summary, then, the following suggestions are offered regarding several possible modes of origin of deep-focus earthquakes:

(1) They may conceivably be due, at least in part, to "explosive chemical inversions at great depth.
(2) They may be due to sudden changes in physical state productive of important volume increases or decreases.
(3) They may be due to slow gas-accumulation followed by sudden gas-eruption.
(4) More probably they are either due to deep tension development along (potential or actual) vertical fissures, or to the development of curved rupture surfaces at less depth, as consequences of the rapid development of basin- or trough-structure in cold, strong rocks, contiguous to the mobile orogenic belts in which rapid deformational movement at shallow depth is now mainly concentrated.

The main factors involved are necessarily the chemical and physical make-up of certain crustal and subcrustal rock-bodies, their forms and mechanical arrangements, their temperatures, the ways in which they are stressed and, above all, the rapidity and irregularity with which stresses are being applied, either by mountain-building movements and processes or otherwise.

Much more remains to be done before we can devise what Father Stechschulte [2] has termed "a complete theory of the Earth's interior, and of its structure, constitution and development", but the writer shares the belief held by many, that by geophysical (and mainly seismic) progression from the geologically known surface to the geologically unknown depths of the Earth, cooperative effort can speedily provide us with far better ideas as to why and how deep-focus earthquakes occur and as to what they signify in terms of the composition and dynamic behavior of our planet.

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A MECHANICAL METHOD OF ANALYZING ACCELEROGRAMS

Frank Neumann

In seismological research, particularly that branch relating to engineering studies and vibration-work, one of the most important practical problems is that of computing the motion of a pendulum when the ground-motion is known. When it is realized that the same formula is also used to estimate the response of engineering structures to earth-movements, its importance is even more apparent. The problem is the reverse of that with which seismologists are ordinarily familiar, namely, that of computing the ground-motion when the pendulum-motion is known from seismographic records. In all computations of this character the assumption of simple harmonic motion makes either phase of the problem quite simple if only rough approximations are desired, but seismological research as applied to engineering problems has imposed new standards on the seismologist and he is faced with many practical problems which heretofore held only theoretical status. As it is now necessary to compute the complete response of a vibrating structure, the assumption of simple harmonic motion no longer suffices and he is confronted with equations which for practical application are so complex that the problems involving them seem almost hopeless of solution.

The equation of motion of a damped pendulum when subject to forced vibration at its point of support is

$$\ddot{y} + 2k\dot{y} + p^2 y = \ddot{x}$$

in which x is the instantaneous displacement of the support, y is the instantaneous displacement of the center of oscillation of the pendulum relative to the moving support, k is a damping factor in which damping is assumed proportion to velocity, and p is a factor involving the pendulum-period. If the motion of the support, x, is known, the solution for instantaneous values of the pendulum-displacement is given by the equation

$$y = (1/u)\int_0^t e^{-k(t - \eta)} \sin \mu\,(t - \eta)\ F\,(\eta)\ d\eta$$

in which $u = (p^2 - k^2)^{1/2}$. This is the equation which must be applied hundreds of times to compute the motion of a vibrating system for a relatively short period of time.

In giving thought to the possibility of using mechanical methods as a solution to the problem, a number seem quite plausible from a theoretical viewpoint. The simplest and evidently most practical method is that involving principles of the simple torsion-pendulum when the ground-motion is given in terms of acceleration. Fortunately acceleration is the easiest of the elements of motion to measure in the case of an earthquake, and accelerometers far outnumber any other type of seismometer in operation by the United States Coast and Geodetic Survey. The underlying principle of the "torsion-pendulum analyzer" is based on the fact that if the torsion-head is turned through angles which are proportional to the instantaneous values of acceleration (as recorded on an accelerogram), the mass of the torsion-pendulum will respond, in angular dis-

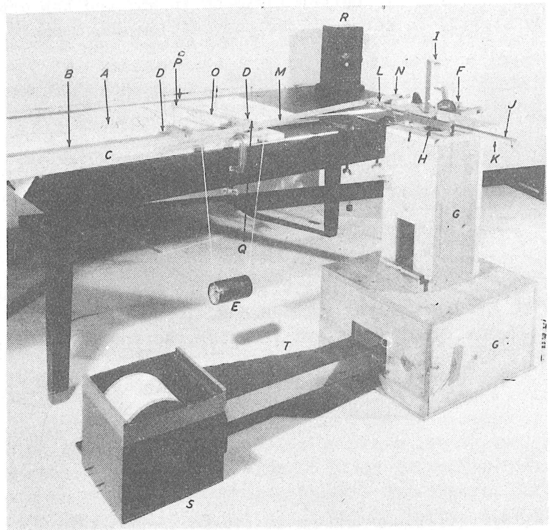

FIG. 1—GENERAL VIEW OF THE TORSION-PENDULUM ANALYZER

Legend for Figure 1

A = aluminum plate for mounting enlarged tracing of original acceleration-curve
B = narrow strip of hard wood fixed to upper edge of A accurately parallel to zero-line of acceleration-curve
C = wide board clamped rigidly to table--upper edge of B slides along a smooth edge of C
D = two gravity-clamps to keep B snugly against edge of C, each clamp consisting of a short strip of wood which presses B against C; short strip is fixed with respect to C except that it is free to move normal to strip and board; another short strip of wood is nailed to top of it to form the stem of a "T" which is restrained in all but longitudinal motion by suitable blocking on C; a pull is exerted on lower end of each "T" by a string attached to a heavy weight; if A is pulled away from C, the pull of the weight is sufficient to force it back, thus keeping the zero-line of acceleration-curve exactly equidistant from sliding surfaces at point of curve being traced
E = 15-pound lead weight operating clamps D D
F = winch used to pull A across the table--this cord is attached to end of B and pulls against restraining forces of D D
G = box enclosing torsion-pendulum
H = torsion-head of the pendulum--only partly visible
I = spring-device to reduce friction in torsion-head by exerting vertical pull
J = rod sliding longitudinally in two sleeve-supports
K = cord attached to J and wound one or two times around H
L = universal joint connecting J with rest of lever-system controlling sensitivity of apparatus
M = lever-arm pivoted at N and connected with J at L
N = pivot-point of M
O = lever-arm pivoted on M at Q, and carrying tracing pointer P at free end; O always moves parallel to J--any slight variation which may enter due to lack of uniform motion of A is negligible
P and Q = pointer and pivot (see O)
R = metronome
S and T = recording apparatus and light-trap T for daytime recording

placement, in a manner identical with that of any other vibrating system having the same period and damping as the torsion-pendulum. It is simply another way of applying acceleration to the support of a pendulum other than by translation of the support, or tilting of the axis of rotation in the case of a horizontal pendulum. The torque in the wire of the torsion-pendulum is equivalent to applying a field of force to the mass of the pendulum, just as tilting a horizontal pendulum furnishes a means of applying a portion of the Earth's gravitational field of force to the pendulum.

The outstanding advantage of the torsion-pendulum lies in the fact that all the motions involved may be slowed up greatly without changing the nature of the response-curve, provided the

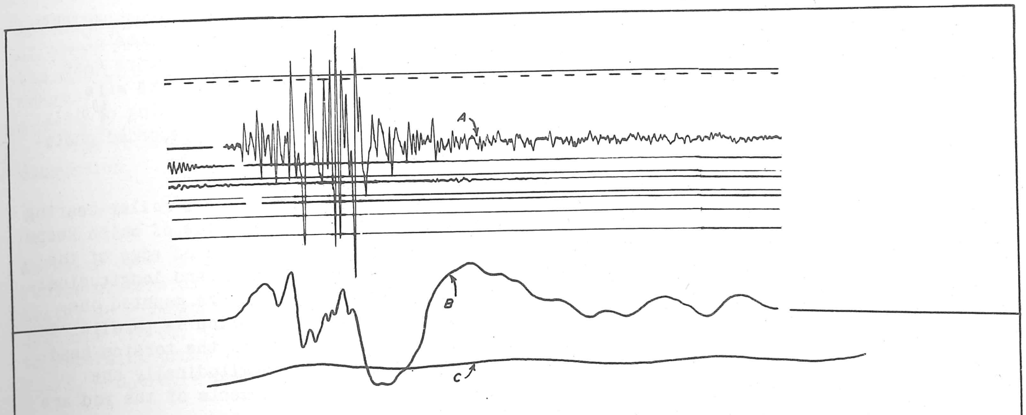

FIG. 2—ACCELEROGRAPH-RECORD OF HELENA (MONTANA) EARTHQUAKE, OCTOBER 31, 1935
(A=PORTION OF RECORD; B=RESPONSE-CURVE OF ANALYZER, EQUIVALENT TO RECORD OF A 5-SECOND PENDULUM MAGNIFYING 4 TIMES; C=CURVE SHOWING STABILITY OF ZERO-POSITION OF TORSION-PENDULUM WHILE IN OPERATION)

speed at which the acceleration is applied and the period of the torsion-pendulum are both slowed up in the same ratio. As torsion-pendulum periods can be made very long without great difficulty, it is possible to apply any observed accelerations to the torsion-head of the pendulum manually,thus doing away with all of the apparatus necessary to reproduce earthquake-motions on true time-scale. It is this characteristic of the torsion-pendulum analyzer which appears to place it above other types in point of practicability. Moreover, it has a sensitivity-characteristic which is ideal. There is almost no limit, in practical work, to the scale of the response-curve. Another advantage is the simplicity of the method of determining instrumental constants. The weakest spot in the apparatus lies in mechanical imperfection, which is not serious, and the instability of long-period torsion-pendulums due to convection-currents and other external causes, all of which can undoubtedly be overcome.

The experimental apparatus (Fig. 1) at the United States Coast and Geodetic Survey has been made in rather crude fashion, and very little work has been done with it, but enough has been accomplished to establish the practicability of the idea. The aim in its design was to reproduce a curve corresponding to that which would be obtained from a 10-second displacement-meter pendulum. Because of lack of time, and another reason to be stated later, the response of a 5-second pendulum was obtained using an acceleration-curve of the Montana earthquake of October 31, 1935. In Figure 2, the portion of the accelerograph-record used is shown at A, the response of the torsion-pendulum at B, and the curve showing the stability of the pendulum on the zero-line at C. Each dash represents a half-second time-interval. In the first attempt at applying acceleration in the form of torsional displacements of the torsion-head, the pendulum-period was slowed up to one-twentieth of the speed of the accelerograph at the time of the earthquake, requiring a torsion-pendulum period of 200 seconds, that is, 20 X 10. This was entirely too fast to follow the acceleration-curve manually with fidelity. It was found necessary to slow up the rate 100 times, thus requiring a pendulum period of 1000 seconds or about 17 minutes. The record shown was made with the pendulum operating at that period and damped magnetically to a ratio of about 8 to 1. However, as the sensitivity or scale of the response-curve decreases as the square of the speed at which the acceleration is applied to the torsion-head, and as it was desired to increase the scale of the response-curve (above unity) because of a certain amount of instability in the zero-line of the apparatus, the accelerations were actually applied at 1/200 normal speed, so that the response was equivalent to that of a 5-second pendulum, with magnification of four instead of unity. In the major characteristics of waves of 5-second period and less, the record checks with the displacement-curve computed by double-integrating the same acceleration-curve, but the presence of a long-period wave (of the order of one minute) on the computed curve makes critical comparison difficult. The response-curve of the analyzer represents the response of a 5-second pendulum and should not be expected to show such a long wave except with greatly reduced amplitude. There is strong evidence that it is present.

A measure of the stability of the zero-line of the apparatus was obtained by tracing over the approximate zero-line of the acceleration-curve at the same rate of speed at which the acceleration was applied to the torsion-head. The response of the pendulum is shown on the illustration at C of Figure 1.

The suspension of the experimental torsion-pendulum is a 30-inch length of steel piano-wire of 0.005-inch diameter. The mass consists of four pieces of aluminum rod 7 inches long projecting horizontally, at equal angles, from a flat aluminum central disc. A one-pound piece of square brass rod is fixed to the outer end of each of the aluminum "spokes." The entire mass weighs a little over four pounds, which is about one-half the breaking strength of the wire. All except the torsion-head of the pendulum is enclosed in a wooden box, the mass being crudely protected against the effects of convection-currents. The motion of the mass is recorded photographically on a recorder.

The torsion-head consists of a brass disc of 1 3/4-inch diameter resting on a roller-bearing unit. The bearing rests in the bottom of a shallow brass basin, the vertical side of which keeps the torsion-head centered as it revolves in the basin. The upper half of the round edge of the disc is spirally grooved. A long narrow rod which is free to move horizontally and longitudinally in two guides is placed within an inch of the torsion-head and at each end are mounted one-inch projections normal to the rod to hold the ends of a tightly drawn cord which is parallel to and moves with the rod. The string, however, is wound one or two times around the torsion-head (in the grooves previously mentioned) so that as the rod and string move longitudinally the torsion-head rotates with very little friction or lost motion. The displacements of the rod are proportional to the instantaneous values of acceleration, the rod being actuated by a lever-system which has at its other end a pointer that is moved manually over the acceleration-curve. As it is necessary that the pointer have only one degree of freedom, the accelerograph-curve must be moved beneath it. This is done by hand, the motion of the aluminum plate on which the curve is mounted being restrained by a pair of gravity-clamps through which a long strip of wood attached rigidly to the top edge of the plate is forced to slide. The strip of wood is, of course, fixed accurately parallel to the zero-line of the curve.

In practice it was found desirable to first place marks on the acceleration-curve at equal time-intervals. In operating the pointer manually one of these marks was passed over at every tick of a nearby metronome.

In practice the fundamental theoretical consideration is: What must the sensitivity of the torsion-pendulum be, in terms of angular deflection of the torsion-head per unit of acceleration, to obtain an equivalent pendulum-response of unit-magnification or some arbitrary magnification? The relation between the various elements involved may be found in a rather simple way by referring first to the ordinary magnification-curve of a damped pendulum for sustained simple harmonic motion which is based on the following equation in which T_0 is the pendulum-period, T_e is the impressed period, and h is the damping factor.

$$V = 1/\left\{[(T_e^2/T_0^2) + 1]^2 + 4(T_e^2/T_0^2)(h^2 - 1)\right\}^{1/2}$$

From this curve it is a well-known fact that a seismograph pendulum with no lever-magnification records practically true displacement for impressed periods which are considerably less than the pendulum-period. If a curve representing the corresponding accelerations is drawn, it represents the sensitivity of the seismograph-pendulum to the equivalent periodic accelerations. For impressed periods which are long relative to the pendulum-period it is well known that the sensitivity of the seismograph to acceleration is constant, and the seismograph is in effect an accelerometer. In the theory of the torsion-pendulum analyzer, however, interest is restricted to the short-period end of the acceleration-curve where the seismograph is functioning as a displacement-meter, and here the equation of the acceleration-curve is practically equivalent to (T_e^2/T_0^2). This means that if periodic accelerations are impressed on the support of a seismograph-pendulum, or on a torsion-pendulum by rotational displacement (of arbitrary magnitude) of the torsion-head, the displacement of the pendulum with respect to the applied periodic acceleration will be in the ratio of T_e^2 to T_0^2. Therefore, if the torsion-head of a torsion-pendulum is given periodic angular motion through an angle θ, which is arbitrarily made equivalent to unit-acceleration, the angular displacement of the pendulum-mass will be (T_e^2/T_0^2) which will be called θ'. If x is the displacement of the light-spot on the recording apparatus which is registering the motion of the torsion-pendulum and L is the distance from pendulum-mirror to recorder, then from the above relations, θ' equals x/2L and

$$x = 2L\ T_e^2\theta/T_0^2$$

It is known that from the equation $a = 4\pi^2 A/T_e^2$, when acceleration is unity

$$A = T_e^2/4\pi^2$$

Therefore, the magnification of the response-curve obtained from the torsion-pendulum is

$$V = x/A = 8\pi^2 L\theta/T_0^2$$

As the angle θ is capable of a great range of values, and is altogether arbitrary, a great range of sensitivity is possible, depending in practice on the mechanical levers used in transmitting the acceleration to the torsion-head, the diameter of which can also be varied through a great range.

The following equation was used for the particular set-up of the torsion-pendulum under discussion

$$V = (8\pi^2 L d_1/r)(d_2 t/T_{oT} d_3)^2$$

in which d_1 = circumferential displacement of torsion-head corresponding to one cm/sec^2; r = radius of torsion-head; T_{oT} = period of torsion-pendulum; t = time-interval between metronome-beats; d_2 = length (in cm) of a one-second time-interval on the magnified trace; d_3 = horizontal distance (in cm) traveled by tracing stylus during one metronome-beat; L = distance from mirror to recording drum; T_o = period of hypothetical pendulum or equivalent seismograph-pendulum; and $T_{oT}/T_o = d_2 t/d_3$ = pendulum-period magnification.

In the experiment the curve used was an accurately drawn tracing about 7.5 times larger than the original accelerogram.

It may be said that the use of the torsion-pendulum in the manner described opens up a new avenue of research in seismological problems. If successful in practice it would seem to eliminate the need for all types of instruments other than accelerometers where quantitative results are of paramount importance. This would include displacement-meters and multi-pendulum analyzers.

The idea might be applied to the design of seismographs outside the ordinary range of periods, especially ultra-long periods, and to seismographs in which moderately long pendulum-periods are desired within a restricted space, especially in portable apparatus. The plan would be to construct a short-period pendulum which would respond as an accelerometer, and have it actuate a torsion-pendulum of any period in (theoretically) the same manner that accelerations are applied to the torsion-head of the torsion-pendulum analyzer. The torsion-pendulum would then be actuated by the earthquake-motion itself through the medium of the accelerometer-pendulum. It is believed that any mechanical difficulties involved could be surmounted. Such a set-up would be especially valuable in serving as a long-period vertical-motion seismometer which would have the same constants as a set of similarly designed horizontal pendulums.

For all set-ups with long-period pendulums the device functions effectively as a double-integrator as evident from the fact that displacement is obtained direct from acceleration in all cases where the torsion-pendulum period is about three times the period of the waves analyzed. As there is almost no limit to the periods obtainable with a torsion-pendulum, it follows that it can be made to function as a double integrator over a great range of wave-periods, or miscellaneous curves if applied in other fields.

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COMMENT ON F. W. SOHON'S PAPER "TRANSFORMING THE STEREOGRAPHIC MAP"

W. L. G. Joerg

The method described in this paper of transforming the zenith of a stereographic map is of importance in its bearing on the construction of such maps for the determination of the location of earthquake-epicenters from seismograph-records. (Although no mention is made in the paper, it is here assumed that its presentation at the joint meeting of two seismological groups envisaged this application of the method.) The stereographic projection is one of two among the group of azimuthal projections (the equidistant, or Postel's, being the other) that are the most appropriate for this purpose, as has been pointed out by a number of writers. As the geographic position of a given seismographic station is made to be the point of tangency of the plane of projection with the sphere, a different grid has to be constructed or computed for each station. In the case of the stereographic projection the grid may be constructed rather simply since the meridians and parallels are arcs of circles. Whether the method proposed in the present paper of deriving one grid from another by changing the zenith offers advantages in time-saving over the conventional method of independent construction is a question it would be of interest to settle by drawing two grids for the same station by the two different methods.

The publications referred to above that deal with the appropriateness of the stereographic and Postel's projections are as follows: R. de Kővesligethy, Comptes-Rendus Réunion Assoc. Internat. de Séismologie, The Hague, September 21-25, 1907, p. 147; Imre Jánosi, Bull. Soc. Hongroise de Géogr., Edit. Internat., v. 37, pp. 35-39, 1909; W. L. G. Joerg, Annals Assoc. Amer. Geographers, v. 2, pp. 49-54, 1912; E. Rudolph and S. Szirtes, Petermanns Mitt., v. 60, I, 1914; J. Howells (not published), Construction of an azimuthal projection for seismographic-station use, paper read at the ninth annual meeting of the Eastern Section of the Seismological Society of America held at Fordham University, New York, April 30-May 1, 1934. On one or the other of these projections published maps exist for the following stations: Budapest (Jánosi, op. cit.); Apia, Samoa (Joerg, op. cit.); and Strasbourg (Rudolph and Szirtes, op. cit.).

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[Note by Father Sohon--The transformation was proposed merely as a mathematical curiosity. In reply to a question proposed by Dr. Hodgson, I expressly stated that I thought it would be faster to recompute rather than to employ the method in a practical problem. I might add, however, that the graphical solution of the problem of finding the true length of the arc of a great circle is a practical application of the method, in which the great circle in question is transformed into the primitive circle. But I propose to eliminate even this construction. The whole purpose was therefore theoretical and had only the intention of giving a better insight into the projection.]