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SYNTHESIS OF WOOD-ANDERSON SEISMOGRAMS FROM BROADBAND DIGITAL RECORDS

By Robert A. Uhrhammer and Eric R. Collins

Abstract

Wood–Anderson torsion seismograms are synthesized by convolution with co-sited broadband digital seismograph recordings and compared with the photographically recorded Wood–Anderson seismograms at Berkeley and Mt. Hamilton. The objective is to use routinely the synthesized seismograms to determine local magnitude. Comparison of local magnitude values calculated from both synthesized and photographic seismograms shows that the variation in local magnitude between the two types of seismograms is less than the variation among the photographic seismograms. Consequently, broadband digital recordings can provide reliable local magnitude estimates without introducing significant bias.

It is shown that the static magnification of standard (T_s = 0.8 sec) Wood–Anderson torsion seismographs, determined from measurement of the free period and tilt sensitivity, is 2080 ± 60, and not 2800 as often reported. This discrepancy is due to the erroneous assumption that the taught-wire suspension in a Wood–Anderson seismograph does not distort significantly, thus leading to an incorrect estimate of the radius of gyration and consequently to a theoretical geometrical magnification of 2800.

Introduction

This study demonstrates how reliably the waveforms and maximum trace amplitudes recorded by "standard ($T_s = 0.8$ sec)" Wood–Anderson type torsion seismographs (hereafter denoted WA) can be synthesized from broadband digital seismographic recordings. The objective is to rely on computer algorithms rather than photographic WA records to determine the local "Richter" magnitude of earthquakes. Such a change in the measurement mode must be calibrated so that there are no discontinuities in the long-term seismic magnitude record.

The University of California Seismographic Stations have operated WA seismographs at the Lick Observatory on Mt. Hamilton (MHC) since 1927 (Byerly and Dyk, 1929) and at two Berkeley sites: at Haviland Hall (BRK) from 1930 (Louderback, 1942) until July 1962 and at Byerly Seismographic Station (BKS) from July 1962 until the present (Uhrhammer, 1989). These seismographs are of importance because they provide the local magnitude (M_L) of regional earthquakes reported in the Bulletin of the Seismographic Stations (routinely since 1948 (Romney and Meeker, 1949)). The accumulation of recordings of recent central California earthquakes on co-sited Wood–Anderson and broadband digital instruments at Mt. Hamilton (MHC) and Berkeley (BKS) now allows a detailed comparison.

Standard ($T_s = 0.8$ sec) Wood–Anderson Type Torsion Seismograph Response

The theory of WA-type torsion seismometers is treated by Anderson and Wood (1924, 1925). The WA instrumental parameters quoted in the manufacturer's technical information and in the literature are: static magnification (V_s) = 2800, free period (T_s) = 0.8 sec and fraction of critical damping (h) = 0.8 (e.g. Richter, 1935; Gutenberg, 1957). F. Lehner tested two WA seismographs on a small shaking table of his own design and found V_s compatible with 2800 within the accuracy of the measurement (Gutenberg, 1957). However, this result is not conclusive as a least-squares fit of the shaking table data given by Gutenberg (1957, Fig. 4) gives $V_s = 2800 \pm 500$.

Calibration of the WA instruments (see Appendix A for procedure), performed after repair or routine maintenance, typically gives a V_s of 2080 ± 60, a T_s of 0.800 sec ± 0.018 sec, and an h of 0.69 ± 0.023 critical (based on calibration of four WA instruments, two at BKS and two at MHC). Detailed calibration results for the WA instruments at BKS and MHC are listed in Table 1. Congealment of the oil in the vibration dampers (see Note 4 in Table 1) is the most common reason for the response to change with time. Representative WA magnification and phase response as a function of frequency are shown in Figure 1. The measured static magnification of the WA seismographs used in the UCB network is significantly lower than 2800. This discrepancy was first noticed circa 1966 while calibrating WA seismographs as part of the seismometry course at Berkeley (T. V.McEvilly, personal comm.) and has since been noted by other researchers (e.g. Bakun and Lindh, 1977; Luco, 1982; Boore, 1989). The shaking table result given above is also compatible with the value of 2080 measured in this paper. 2800 is the theoretical geometrical magnification (G) (Anderson and Wood, 1925) as shown in Appendix B. We infer that the 2800 static magnification value for the WA commonly quoted is in fact the theoretical geometrical magnification determined by the manufacturer and not the static magnification determined from measurement of the natural period and tilt sensitivity.

The observation that the static magnification is different than the theoretical geometrical magnification implies that the Zollner taught-wire suspension system used in the WA is distorting in a manner that increases the moment of inertia about the suspension axis. Wood and Anderson (1925) claim that distortion in the Zollner suspension is negligible. Analysis of the forces acting on the suspension, however, shows that the longitudinal axis and the center of mass of the asymmetrical inertial mass can deviate sufficiently from the rotation axis to increase the moment of inertia about the rotation axis by a factor of approximately two (see Appendix B). This change is sufficient to lower the theoretical geometrical magnification to approximately 2080 and resolve the discrepancy.

TABLE 1

WA CALIBRATION PARAMETERS

Station	Component	T_s (sec)	Static Magnification	Damping
BKS*	SN¶	0.804 ± 0.007	2060 ± 36	0.656 ± 0.003
BKS*	WE	0.798 ± 0.019	2110 ± 95	0.713 ± 0.007
MHC†	SN	0.814 ± 0.008	2180 ± 52	0.708 ± 0.014
MHC†	EW	0.799 ± 0.005	2000 ± 28	0.680 ± 0.025
MHC‡	SN	0.767 ± 0.034	2100 ± 192	0.668 ± 0.026
MHC‡	EW	0.614 ± 0.010	2120 ± 115	0.574 ± 0.084
MHC§	SN	0.804 ± 0.005	2010 ± 26	0.700 ± 0.032
MHC§	EW	0.769 ± 0.006	2030 ± 44	0.710 ± 0.034

* Calibrated May 1989.

† Calibrated November 1983.

‡ Calibration checked June 1989; the low T_s and damping and relatively large standard errors are attributed to presence of congealed oil in the upper vibration dampers.

§ Cleaned, oiled, and recalibrated June 1989.

¶ First direction is up on seismogram.

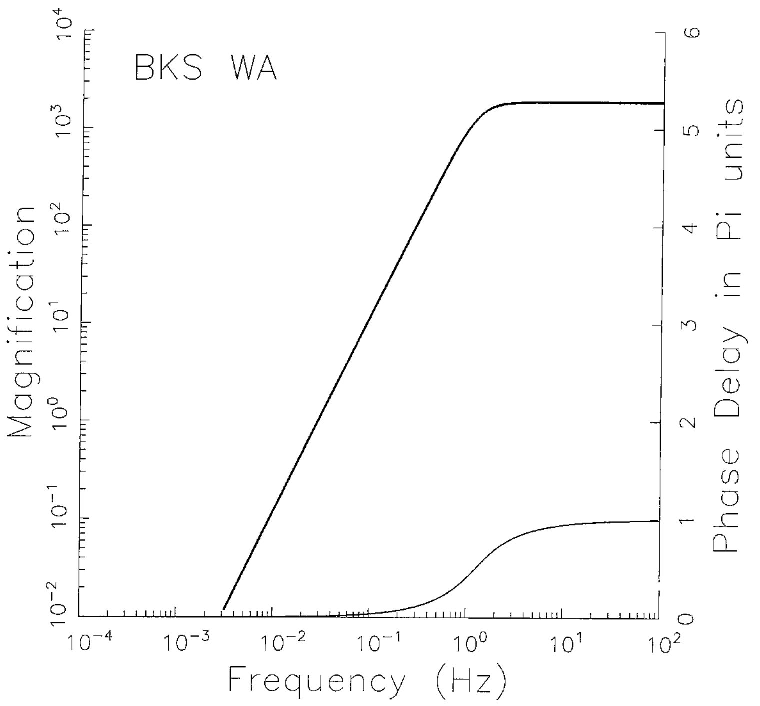

FIG. 1. BKS WE-component WA magnification and phase response as a function of frequency.

Because the M_L scale is defined using the maximum trace amplitude recorded on WA seismograms (Richter, 1935; Gutenberg and Richter, 1942), a static magnification closer to 2080 than 2800 does not matter when only WA instruments are used in the calculation. However, when synthesizing WA seismograms, assuming a static magnification of 2800 (as has been common practice, e.g., Bakun and Lindh, 1977; Kanamori and Jennings, 1978; Luco, 1982) will lead to a systematic over estimation of M_L by an average of 0.13 M_L units.

The damping value of the WA instrument may also affect M_L determination. The damping parameter measured at BKS and MHC is 0.70 ± 0.023 critical (see Appendix A) rather than the 0.80 critical quoted by Richter (1935). This difference does not generally bias significantly the maximum trace amplitudes which usually occur at periods well below the natural period ($T_s = 0.8$ sec) of the seismometer. For example, in the case of a monochromatic wave at $T = 0.8$ sec and a damping of 0.70 critical, the maximum trace amplitude will be increased by only 14 per cent over a WA with a damping of 0.8 critical which is equivalent to a maximum M_L bias of +0.06.

Another source of bias in the determination of M_L is variation in $-\log A_o$ ($M_L = \log A - \log A_0$) as a function of region and distance (Bakun and Joyner, 1984; Hutton and Boore, 1987). In this paper, $-\log A_0$ from Richter (1935) as modified by Gutenberg and Richter (1942) for distances less than 30 km is used to determine M_L. Station adjustments of 0.0 for BKS and +0.1 for MHC (Gutenberg and Richter, 1942) are also included in the calculations. Note that for the comparison of M_L's, determined using maximum trace amplitudes from co-sited WA and SWA instruments, it does not matter which $-\log A_0$ scale is used. However, for the absolute determination of M_L, one must specify which $-\log A_0$ scale is being used (especially at distances less than 30 km).

DIGITAL SEISMOGRAPH RESPONSE

Two different types of seismographs in the Berkeley Digital Seismograph Network (BDSN) are available for comparison. The broadband three-component instrumentation at BKS consists of a set of ultra-long-period (ULP) Sprengnether S-5100 seismometers operating at an inertial free period of 100 sec. At MHC there is a set of very-broadband (VBB) Streckeisen STS-1 seismometers operating under a force feedback with an equivalent free period of 360 sec. The velocity proportional outputs (flat from 10 mHz to 5 Hz at BKS and 2.7 mHz to 5 Hz at MHC) are anti-alias filtered with 10-pole Butterworth lowpass filters at 5 Hz and digitized at a rate of 20 samples per second per channel with a 16-bit digitizer and no gain-ranging (Bolt *et al.*, 1988). Sensitivity and phase response curves for the ULP at BKS are shown in Figure 2.

As a check on the accuracy of calibration, a set of Guralp CMG-3 extended broadband (EBB) seismometers were co-sited in 1989 with the ULP's at BKS. Both sets of instruments were carefully calibrated and their respective displacement transfer functions computed (Technical Staff, 1989). Ground displacement seismograms for recorded earthquakes were then obtained by deconvolution of the respective transfer functions. The resulting ground displacement records matched exceptionally well, usually within 1 per cent in the 10 mHz to 5 Hz passband common to both instruments. The largest differences were less than 5 per cent at the 10 Hz Nyquist frequency.

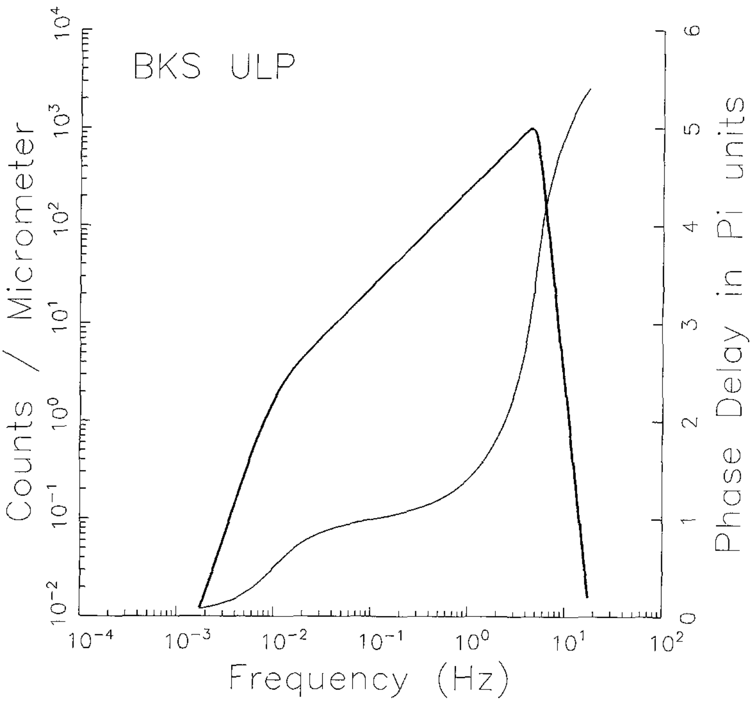

FIG. 2. BKS ULP displacement transfer function showing the sensitivity (thick line) and the phase response (thin line) as a function of frequency. A + 1 slope in displacement response from 10 mHz to 5 Hz is the same as a "flat" velocity response.

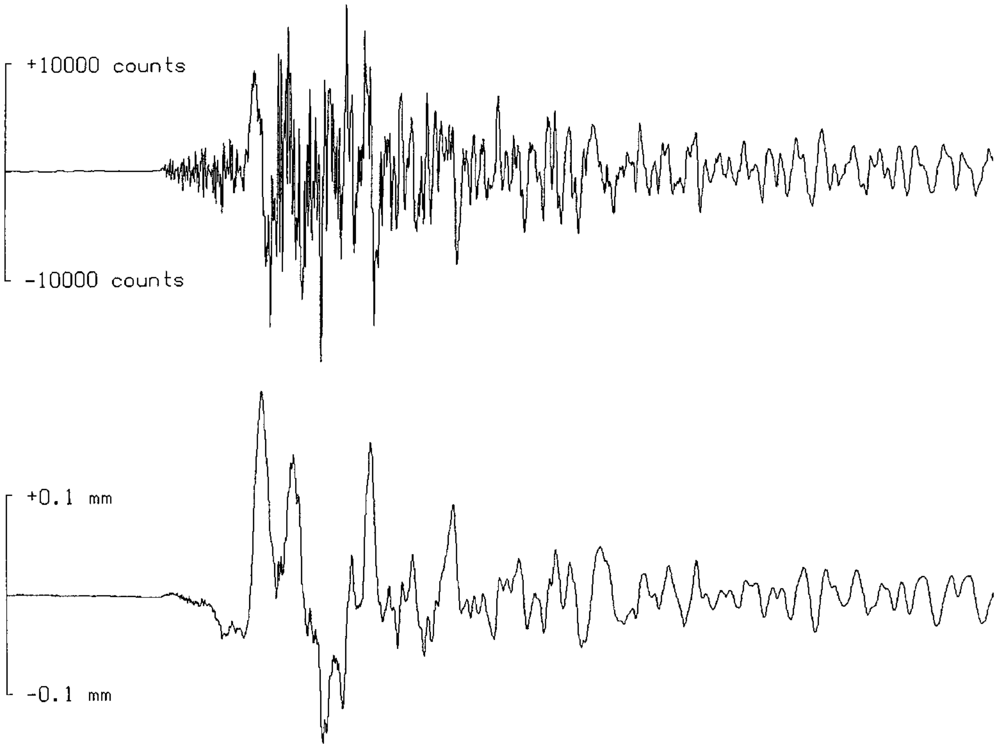

FIG. 3. 100-sec time histories of the "raw" digital data (*upper*) and the corresponding ground displacement (*lower*) for the BKS WE-component (west up) record of the M_L 4.7 Alum Rock earthquake (no. 10 in Table 2 and Fig. 4);

The absolute calibration accuracy of the Streckeisen VBB seismometers at MHC was checked by determining the sensitivity of the NS component to tilt. A 3-microradian resolution tilt table was constructed from a Sprengnether LP horizontal baseplate with the addition of a half-inch aluminum plate large enough to mount the Streckeisen and with the addition of a protractor with a pointer on the large (rear) leveling screw. The NS component Streckeisen (SN 58406) was tilted through a 160-microradian range in 20-microradian steps and the resulting sensitivity of (372 ± 10) volts/gal is not significantly different from the value of 375 volts/gal on the calibration sheets supplied by Streckeisen. This instrument has operated since April 1985 with essentially no change in its response in the 0.1 Hz to 5 Hz passband. (The long-period response changed in October 1987 with the addition of the VBB modification which changed the effective free period of the seismometer from 20 sec to 360 sec.)

An example of a "raw" broadband digital data record and the corresponding ground displacement record registered by the ULP seismograph at BKS is given in Figure 3. The "raw" digital data are flat in velocity response from 10 mHz to 5 Hz as shown in Figure 2 and the ground displacement record is flat from the 10 mHz seismometer corner to the 10 Hz Nyquist frequency.

THE ADOPTED DATA SET

Broadband instruments have been co-sited with WA instruments since October 1986 at MHC and since May 1987 at BKS (Uhrhammer, 1989). Numerous regional earthquakes have been recorded on-scale simultaneously by both types of instruments. Sixteen earthquakes (see Table 2 and Fig. 4), chosen for the comparison, sample a wide range of distances, azimuths, local magnitudes, and maximum trace amplitudes as shown in Tables 2, 3, and 4. Selected MHC events are limited to those occurring after the late June 1989 calibration of the WA (see Table 1, note 5).

SYNTHESIS OF STANDARD WOOD–ANDERSON TYPE SEISMOGRAMS

WA seismograms are synthesized from the digital records by frequency domain convolution. The procedure is: deconvolve the broadband instrument transfer function to obtain a ground displacement record; lowpass filter (typically a four-pole Butterworth amplitude filter at 7.5 Hz) if required by a low signal-to-noise ratio; convolve the WA transfer function; and plot at the 300 dot per inch plotter resolution using five-point Lagrangian interpolation to obtain a synthesized WA

TABLE 2

EARTHQUAKE LOCATION AND MAGNITUDE

Earthquake Number	Date (dd/mon/yy)	Time UTC	Latitude (°N)	Longitude (°W)	Depth (km)	M_L(n)	Location
1	26Jul88	0326	36.56	121.18	3.3	4.7 (10)* (0.18)†	Bear Valley
2	27Jul88	0358	37.88	122.27	8.2	1.9 (2) (0.10)	Berkeley
3	19Sep88	0256	38.52	118.36	9.1	5.3 (6) (0.18)	Hawthorne, NV
4	26Oct88	0750	37.88	122.19	5‡	1.1 (2) (0.06)	Berkeley Hills
5	16Dec88	0150	36.83	121.27	8.2	3.8 (4) (0.11)	Hollister
6	30Dec88	2354	37.29	121.69	6.8	4.3 (6) (0.08)	San Felipe Valley
7	17Jan89	2239	37.81	122.60	11.8	2.5 (4) (0.08)	San Francisco
8	08Feb89	1629	37.42	121.75	6.2	2.3 (4) (0.22)	Alum Rock
9	03Mar89	0703	34.72	127.95	5‡	4.9 (2) (0.15)	San Luis Obispo
10	03Apr89	1746	37.42	121.80	8.5	4.7 (4) (0.20)	Alum Rock
11	22Jun89	2115	37.28	116.41	0‡	5.3 (8) (0.15)	Nevada Test Site
12	11Jul89	0413	37.42	118.64	11.6	4.5 (4) (0.22)	Bishop Area
13	18Jul89	1107	36.91	121.35	6.8	3.6 (4) (0.07)	Hollister
14	24Jul89	0237	36.80	121.55	5.8	3.2 (4) (0.09)	Hollister
15	24Jul89	0308	36.81	121.56	5.9	3.2 (4) (0.10)	Hollister
16	31Jul89	1025	37.47	121.78	5.3	3.1 (4) (0.11)	Calaveras Reservoir

* Number of observations used to compute M_L.

† M_L standard error.

‡ Depth restrained in solution.

TABLE 3

BKS DATA

Earthquake Number	Delta (km)	Azimuth (degrees)	Cmp	WA (mm)	SWA (mm)	log (SWA/WA)
1	173	147	SN	14.7	14.87	0.005
			WE	11.2	10.39	−0.033
2	3	270	SN	2.5	1.68	−0.173
			WE	3.5	1.84	−0.279*
3	347	77	SN	13.0	11.17	−0.066
			WE	16.8	16.46	−0.009
4	4	90	SN	0.5	0.26	−0.284*
			WE	0.6	0.37	−0.210*
5	144	147	SN	3.8	4.01	0.023
			WE	2.7	2.89	0.030
6	82	144	SN	22.0	19.35	−0.056
			WE	23.0	21.88	−0.021
7	33	257	SN	2.6	1.37	−0.278*
			WE	2.0	1.64	−0.086
8	67	140	SN	0.5	0.38	−0.119
			WE	0.5	0.50	0.000
9	622	237	SN	0.7	0.63	−0.046
			WE	0.6	0.57	−0.022
10	63	143	WE	55.6	52.28	−0.027

* See text for explanation of these large negative values.

TABLE 4

MHC DATA

Earthquake Number	Delta (km)	Azimuth (degrees)	Cmp	WA (mm)	SWA (mm)	log (SWA/WA)
11	463	272	SN	7.0	8.02	0.059
			EW	4.2	5.04	0.079
12	267	268	SN	6.8	8.09	0.075
			EW	4.1	4.80	0.068
13	54	331	SN	5.0	5.59	0.048
			EW	6.5	6.77	0.018
14	60	352	SN	2.0	2.30	0.061
			EW	2.6	3.05	0.069
15	60	352	SN	2.3	2.72	0.073
			EW	2.7	3.15	0.067
16	18	139	SN	16.7	19.21	0.061
			EW	26.9	25.51	−0.023

seismogram (hereafter denoted by SWA). The convolution method assumes that the WA instrument is adequately modelled by a linear transfer function, which will be discussed later. This frequency domain approach was also used by Bakun *et al.* (1978) in the synthesis of WA seismograms from short-period seismograph records.

Consider the BKS recording of the M_L 4.7 earthquake which occurred on 3 April 1989 at Alum Rock along the Calaveras fault 63 km SE of Berkeley (see Fig. 4 and event 10 in Tables 2 and 3). BKS WE-component (west is up on record) raw data and ground displacement and SWA records are shown in Figure 3 and 5a, respectively. The ground displacement signal-to-noise ratio exceeds 40 db over a 0.6 Hz to 6 Hz frequency band as shown in Figure 6. Note that the raw data in Figure 3 are about 33,000 counts peak-to-peak and that the corresponding synthesized WA

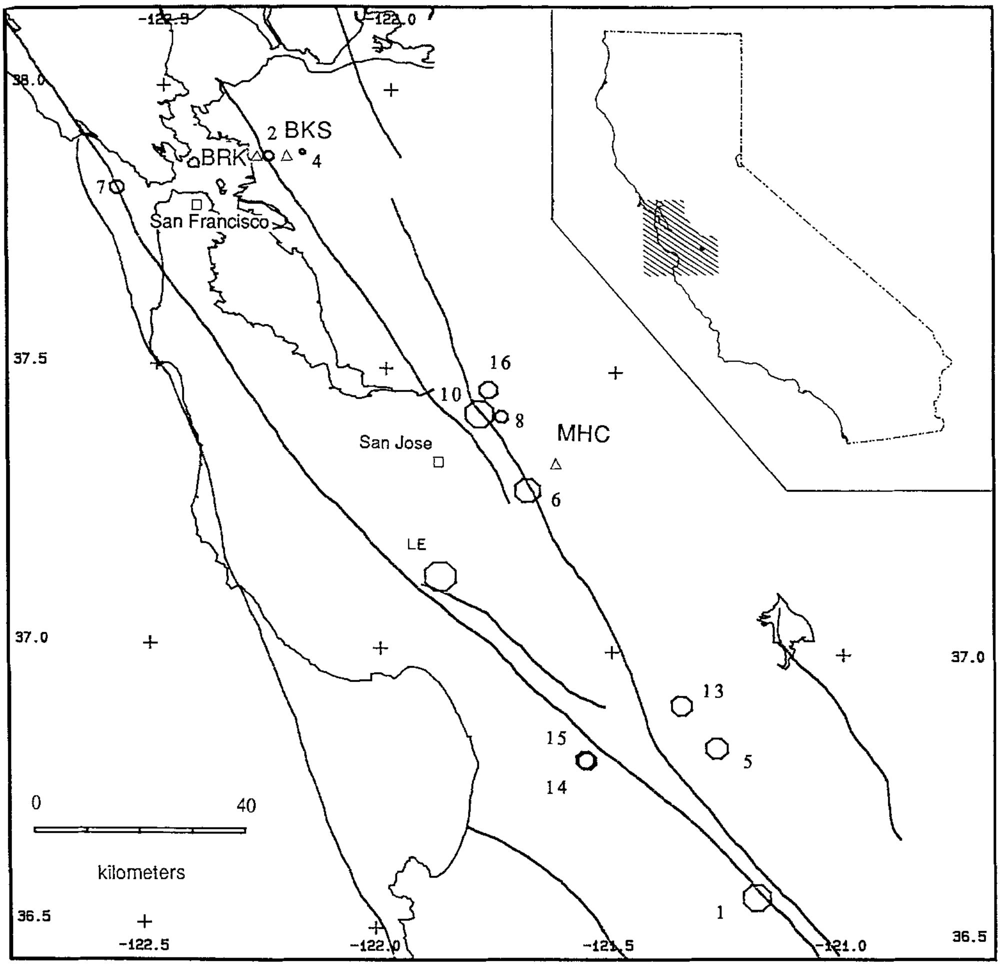

FIG. 4. Map of the central coast region of California showing the locations of BKS, BRK, and MHC and the epicenters of earthquakes studied in this paper. The octagonal symbols representing epicenters are scaled to M_L with the smallest M_L 1.1 and the largest M_L 5.3. The numbers beside the symbols correspond to Table 2 and LE shows the locations of the Lake Elsman earthquake. Number 10 is the Alum Rock epicenter.

record maximum trace amplitude is 55.6 mm (Fig. 5a and Table 3). The BDSN system clipping level is 65,536 counts peak-to-peak and the maximum WA trace amplitude which can be synthesized from this system is approximately 110 mm. Thus, for earthquakes with an amplitude spectrum similar to that shown in Figure 6, the maximum size earthquake which can be recorded on-scale at a distance of 63 km is approximately M_L 4.8. For comparison, the largest maximum trace amplitudes readable on photographic WA records rarely exceed 110 mm.

With broadband and other types of digitally recorded seismographic data (e.g., from strong-motion accelerometers), it is possible to synthesize WA records where the maximum trace amplitudes used to determine M_L are much larger than the maximum which can be recorded on-scale on a standard WA seismogram (Kanamori and Jennings, 1978; Roca, 1982). The maximum trace amplitudes recordable on a standard WA seismogram are limited to approximately 140 mm by the aperture of the drum lens. For WA maximum trace amplitudes of 140 mm or less, the cross-

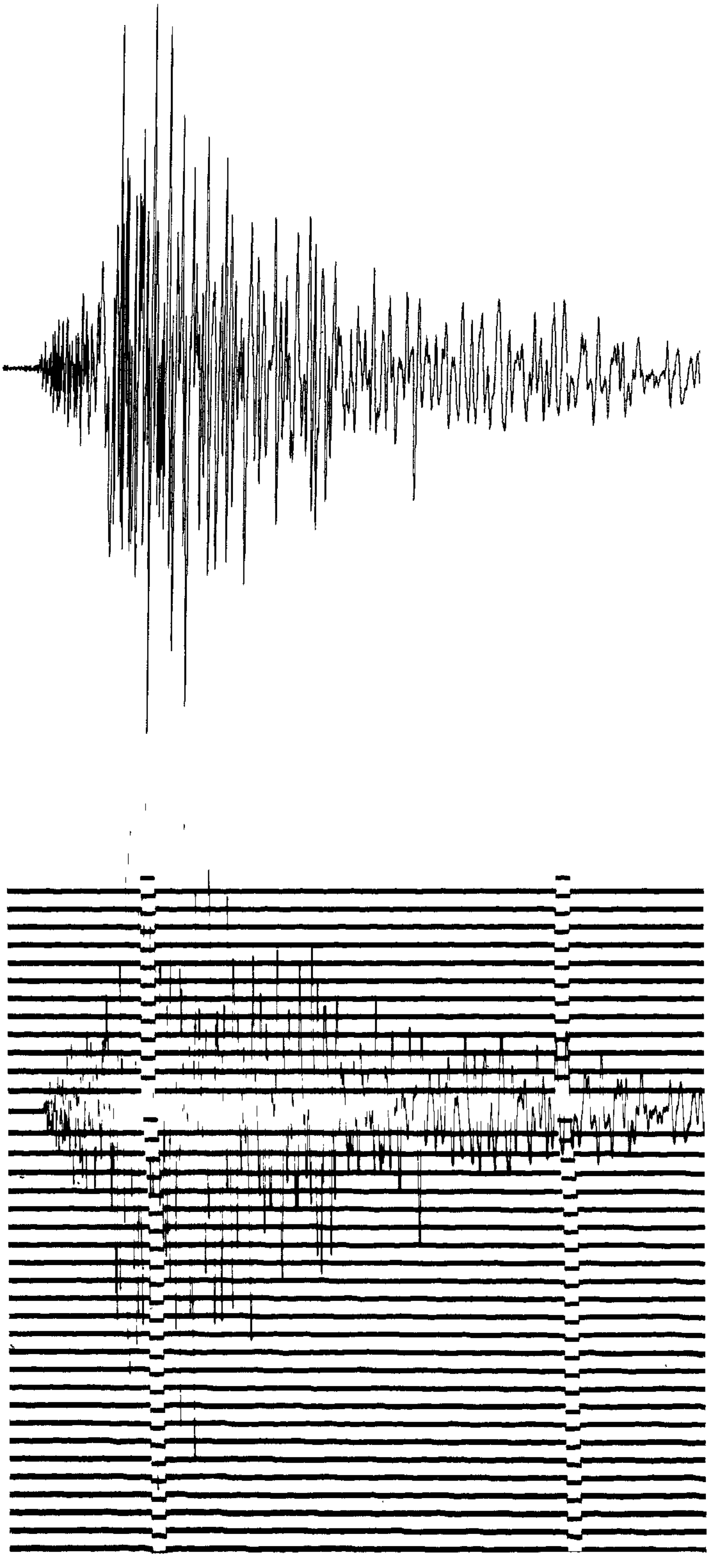

FIG. 5. (a) Synthesized WA seismogram (upper trace) and (b) photographic WA seismogram for the BKS WE-component of ground motion (west up on records) for the Alum Rock earthquake (no. 10 in Table 2). The minute marks on the photographic record are 60 mm apart and the maximum trace amplitude is 55.6 mm. For ease of comparison, it is helpful to make a transparency of the synthetic seismogram and overlay it on the photographic seismogram. Note the largest differences occur in the beginning of the S-wave coda where the highest frequencies are present. For the latter 75 per cent of the record (including the maximum trace amplitudes) the differences are within the range expected given the uncertainty in the calibration of the WA instrument.

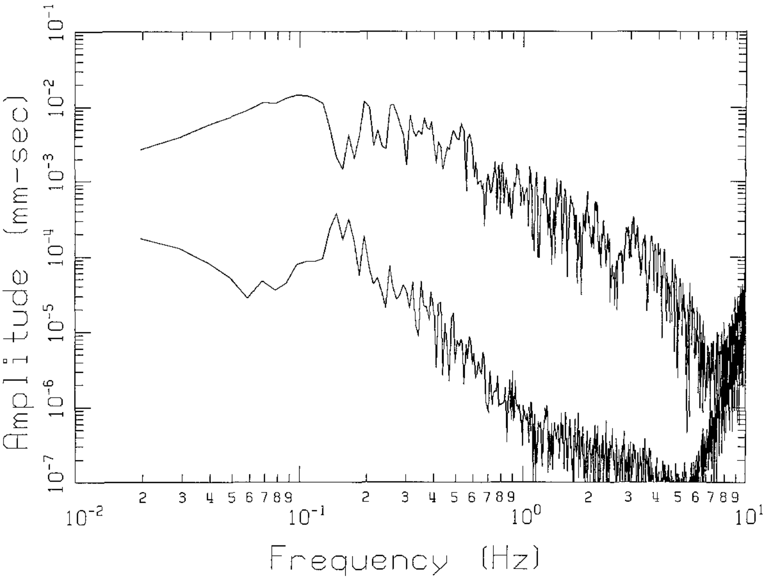

FIG. 6. Fourier amplitude plot of the signal (*upper*) and noise (*lower*) spectra over the 10 mHz to 10 Hz frequency band for the ground displacement data in Figure 3. The sharp increase in the background noise spectra above 5 Hz is due to deconvolution of the 10 pole anti-aliasing filter. The noise spectrum is for 100 sec of ground motion immediately preceding the earthquake.

axis coupling (sensitivity to orthogonal component of ground motion) is negligible (less than 3.5 per cent) and the WA response is a linear function (to second order) of the ground motion. Richter's (1935) definition of M_L implicitly assumes the response of a WA seismograph is linear. Thus it is valid to calculate M_L from maximum trace amplitudes much larger (up to 100+ times, say) than can be observed on a standard WA seismogram. As an example, the SWA maximum trace amplitudes for the Santa Cruz Mountains earthquake of 18 October 1989 (0004 UTC), determined from strong-motion accelerometer records on rock sites at Berkeley, Rincon Hill and Yerba Buena Island (at distances of 100 km, 96 km, and 95 km, respectively), are A_N = 6860 mm, 10800 mm, and 4810 mm, and A_E = 17300 mm, 13800 mm, and 12400 mm, respectively, and the inferred local magnitude is 7.0 ± 0.08. For comparison, the local magnitude determined from the 100× torsion instruments at Berkeley (BRK) is 7 and the NEIC surface-wave magnitude is 7.1.

COMPARISON OF SYNTHESIZED AND PHOTOGRAPHIC WOOD–ANDERSON TYPE TORSION SEISMOGRAMS

A visual comparison of corresponding BKS photographic and synthesized WA records shows that differences in the two types of records are not easily detected when the maximum trace amplitudes are under 10 mm. Figures 5a and 5b illustrate how closely a photographic WA and the corresponding synthesized WA record match. This comparison was chosen because of the high signal-to-noise ratio, the quality of the photographic WA record, and some differences are readily discernable. At small amplitudes and at low frequencies (i.e., in the latter part of the S-wave coda) the two traces match within 3 per cent on peaks. At maximum trace amplitudes the peaks match within 6 per cent. The most noticeable differences are in the first 5 sec of the high-frequency (8 Hz to 12 Hz) portion of the S-wave coda (about 3 sec prior to the minute mark) where the corresponding amplitude peaks vary by as much as 35 per cent. This large variation is attributable to the limited high-frequency response of the SWA due to the 20 Hz sampling rate. The maximum trace amplitudes, generally observed in the 1 Hz to 5 Hz range, are not usually affected by this sampling rate limitation. None of the differences between these records, however, are larger than expected given the uncertainties in the WA calibration (see Table 1) and the limitation in modeling high frequencies due to the SWA 10 Hz Nyquist frequency.

Coherency between BKS SWA and high-gain torsion (WA) records was calculated to check for frequency dependence between the two types of records. The high-gain torsion signals, recorded on analog magnetic tape (Collins *et al.*, 1989), were used because they have the same frequency response as a standard WA and are easily digitized. The coherency is typically greater than 0.99 from 0.25 Hz to 6.4 Hz and greater than 0.95 from 0.19 Hz to 8.3 Hz. Thus there is no significant variation in frequency response between SWA and WA records in the frequency band where the maximum trace amplitudes, used to calculate M_L, are generally observed.

The measured differences between the photographic WA and synthesized WA maximum trace amplitudes used to calculate local magnitude (M_L) for BKS and MHC are tabulated in Tables 3 and 4 and plotted in Figure 7. The vertical axis in Figure 7 is equivalent to the difference in calculated M_L. The largest negative log(SWA/WA) values in Figure 7 are for two small earthquakes ($M_L < 2$) located within 4 km of BKS (no. 2 and 4 in Table 3). The WA seismograms for these two earthquakes contain high frequency energy (above 8 Hz) and the corresponding SWA seismograms have a low signal-to-noise ratio. The combination of significant high-frequency energy and poor signal-to-noise ratio leads to unreliable estimates of log(SWA/WA). Neglecting these outliers, the average log(SWA/WA) for BKS is −0.013 ± 0.036 while for MHC it is 0.062 ± 0.017. For comparison, the standard error in M_L, for 233 regional earthquakes ($M_L \geq 2.5$) analyzed by station personnel during 1988, is 0.19 and the range of standard error in M_L given in Table 1 is thus typical. The inference is that the inherent variation in estimates of M_L from the

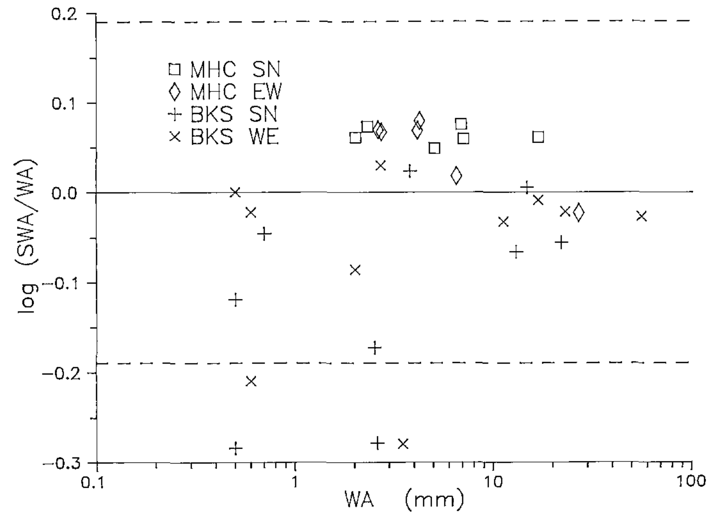

FIG. 7. Plot of Log(SWA/WA) versus Log (WA) for all observed data in this study. SWA and WA are the maximum trace amplitudes measured on the synthetic and photographic seismograms, respectively. The dashed lines, for comparison, show the typical standard error in M_L calculated from WA seismograms (see text). Note that the points for MHC average about 0.06 higher than BKS. The data with the larger deviations from zero are discussed in the text.

photographic records is significantly larger than the differences in the M_L estimates between the synthesized and photographic records.

DISCUSSION AND CONCLUSIONS

As well as reducing response time, the principal advantage of synthesizing WA seismograms from digital instruments lies in the ability to obtain a more representative M_L determination by using several telemetered stations. A group average is particularly useful when strong ($M_L > 5$) regional earthquakes occur that have significant directivity and/or propagation path effects which can bias a single-station M_L determination. As an example, the 27 June 1988 Lake Elsman earthquake (LE in Fig. 4) which occurred on the San Andreas fault 27 km south of San Jose had a 5.7 M_L measured at Berkeley (BKS; 90 km to the north) and a 4.7 synthetic M_L measured at the San Andreas Observatory (SAO; 56 km to the southeast) and a M_L of 5.3 using all available UCB WA records (Collins *et al.*, 1989). In this case, the determination of M_L using only BKS was 0.4 M_L higher than the average for the Berkeley network and also higher than coda duration magnitude 5.0 determined by the USGS in Menlo Park. When the trace amplitudes are large, the synthesized WA seismograms have a second advantage over the photographic WA seismograms, in that the maximum trace amplitude, readily identified on the synthesized records, may be missed on the photographic WA seismograms because the trace becomes too faint to be detected.

In order to synthesize WA seismograms from BDSN or other types of digital records, it is necessary to have an accurate transfer function for the WA instruments to obtain an unbiased estimate of M_L. Based on the calibration of WA instruments at BKS and MHC, we recommend that $V_s = 2080$, $T_s = 0.8$ sec, and $h = 0.7$ critical be used to calculate the transfer function of the "standard ($T_s = 0.8$ sec)" WA torsion seismograph. Note that these values differ from the commonly published values $V_s = 2800$, $T_s = 0.8$ sec, and $h = 0.8$ critical in that V_s is reduced 26 per cent to comply with the measured tilt sensitivity and h is set to approximately 0.7 to expedite field calibration.

The average logarithmic difference between the corresponding SWA and WA maximum trace amplitudes at BKS (-0.013 ± 0.036) is not significantly different from zero which shows that the SWA provide an unbiased estimate of M_L. The corresponding difference at MHC ($+0.062 \pm 0.017$) is significantly different from zero at the 95 per cent confidence level. However, this difference will not alter significantly the inherent variation in M_L based on WA recordings.

Caution is advised when calculating M_L from SWA records where the epicentral distance is less than about 30 km. At short distances two problems arise. First, there can be significant energy in the spectrum above the Nyguist frequency of the digital record which will lead to an underestimate in the maximum trace amplitudes used to calculate M_L. Second, there is significant variation in the value of $-\log A_0$ as a function of location and distance which leads typically to an overestimate of M_L. We recommend that SWA records at distances of less than 30 km not be used to routinely calculate M_L.

We conclude that SWA records derived from a regional network of broadband digitally recording seismographs can successfully replace the photographic WA records for the purpose of routinely calculating M_L without introducing significant bias in the long-term seismicity record.

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SEISMOGRAPHIC STATION
475 EARTH SCIENCES BUILDING
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720

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APPENDIX A: WA CALIBRATION PROCEDURE

The photographically recording WA seismographs installed at MHC and BKS were manufactured by the Fred C. Henson Co. and Lehner and Griffith Instruments, respectively. The calibration procedure for these WA instruments consists of five steps:

1. adjust the free period (T_s) to 0.8 sec, with the damping magnet removed;
2. determine the tilt sensitivity, with the damping magnet removed, by tilting the seismograph through a known angle (a) (typically 300 microradian) and measuring the resulting deflection (d) of the light spot (typically 100 mm);
3. replace the damping magnet and adjust the tilt sensitivity to the same value as in step 2 by rotating the period adjustment foot;
4. adjust the magnetic damping to give an overshoot ratio of approximately 20 to 1 which is equivalent to a damping of about 0.7 critical (a 20 to 1 ratio is used in place of the 50 to 1 ratio required to achieve 0.8 critical damping in order to expedite adjustment at remote station sites as it can be set approximately without having to develop the photographic seismograms); and
5. repeat steps 3 and 4 as needed until both the damped tilt sensitivity and the fraction of critical damping are satisfactory.

The last three steps are necessary because magnetic contamination of the inertial mass can significantly alter the free period of the seismometer in the presence of the damping magnetic field (Neumann, 1926a and b; Lehner and Griffith, 1954). Setting the tilt sensitivity to the value determined in step 2, when the damping magnetic field is present, is equivalent to adjusting the free period to 0.8 sec.

As an example the static magnification (V_s) for a $d = 100$ mm (10 cm) deflection when subjected to a tilt of $a = 300$ microradian is:

$$V = \frac{39.48\ d}{T_s^2 g a} = 2100$$

where $g = 980$ cm/sec^2.

APPENDIX B: ANALYSIS OF WA THEORETICAL GEOMETRICAL MAGNIFICATION

Anderson and Wood (1925) calculated the principal moment of inertia (I_{zz}) and radius of gyration (k_z) (center of oscillation) of the WA-type torsion instrument by assuming that the 0.944 gram copper inertial mass is represented by a cylinder which is rotating about an edge coaxial to the suspension (z) axis (i.e., they assumed that the distortion in the Zollner-type wire suspension is negligible). The Lehner and Griffith (UED TS-220) WA-type seismometer uses a 3.5-cm-long copper cylinder 0.2 cm in diameter so $I_{zz} = 0.0148$ gm-cm^2 and $k_z = 0.150$ cm ($\frac{3}{2}$ of radius). The optical lever arm is 400 cm (measured 4 × 100 cm including doubling mirror), and thus the theoretical geometrical magnification (G) for the WA-type instrument is 400/0.150 = 2667. Taking into account the asymmetry of the inertial mass and attached mirror, we find $I_{zz} = 0.0193$ gm-cm^2, $k_z = 0.143$ cm, and $G = 2800$ (as commonly quoted in the literature).

The components of the inertia tensor (see Symon, 1960, section 10-5) in gm-cm^2 derived from the mass dimensions for the Lehner and Griffith instrument (assuming a non-distorting suspension) are:

$$\begin{array}{lll} I_{xx} = +0.9913 & I_{xy} = -0.0019 & I_{xz} = +0.0038 \\ I_{yx} = -0.0019 & I_{yy} = +0.9778 & I_{yz} = +0.0313 \\ I_{zx} = +0.0038 & I_{zy} = +0.0313 & I_{zz} = +0.0193 \end{array}$$

where the coordinate system is right-handed with the z axis coaxial to the suspension axis and the y axis in the direction of the center of mass from the suspension axis. Note that the off-axis principal moments of inertia (I_{xx} and I_{yy}) are some 50 times larger than the principal moment about the suspension axis (I_{zz}) and also that the product of inertia (I_{yz}) is larger than I_{zz}. The implication is that a relatively small distortion in the Zollner-type wire suspension will have a significant effect on the I_{zz} principal moment of inertia.

The force-couples acting on the inertial mass, due to inclination of the suspension axis under gravity distort the 21-cm-long, 0.8 mil tungsten (Bulk modulus = 2.4×10^{12} dynes/cm^2) suspension wire. The inertial mass is both rotated (see Morse and Feshbach, 1953, section 1.6) and translated (see Symon, 1960, section 10-5) in the midsagittal plane relative to the suspension points by approximately $-0.02°$ and 0.03 cm, respectively. The inertia tensor about the suspension points in this configuration is:

$$\begin{array}{lll} I_{xx} = +0.9661 & I_{xy} = -0.0019 & I_{xz} = +0.0038 \\ I_{yx} = -0.0019 & I_{yy} = +0.9363 & I_{yz} = +0.0310 \\ I_{zx} = +0.0038 & I_{zy} = +0.0310 & I_{zz} = +0.0355. \end{array}$$

Note that the I_{zz} component has approximately doubled in size which leads to a radius of gyration (k_z) of 0.194 cm and a corresponding theoretical geometrical magnification of 2060. This agrees with the range of static magnification (2080 ± 60) measured by tilting the WA seismometers.

An independent method for detecting significant distortion in the 0.8 mil suspension wire is to measure the inclination (i) of the suspension points from vertical and the sensitivity of the free period to changes in the inclination angle (dT/di) for a Lehner and Griffith WA-type instrument operating at a free period (T_s) of 0.8 sec. Assuming that the suspension wire does not distort and that the restoring torques due to rigidity of the wire and magnetic contamination of the inertial mass are negligible, upper bounds on the allowable inclination and sensitivity of free period to inclination angle at a T_s of 0.8 sec are 0.51° and −0.78 sec/degree, respectively. Including the restoring torques due to wire rigidity and magnetic mass contamination in the calculation will decrease these upper bounds.

The measured inclination of a Lehner and Griffith seismometer operating at a 0.89 sec T_s is $0.87° - 0.09°$. This value was determined using an accelerometer, with a sensitivity of 5 mV/gal, to measure the tangential component (perpendicular to the suspension housing) of the gravitational restoring force and by using vernier calipers to determine the coaxial deviation of the suspension points relative to the suspension housing. The corresponding measured dT/di is −0.65 sec/degree ± 0.05 sec/degree. Both i and dT/di are significantly outside the allowable bounds for a rigid suspension at more than the 95 per cent confidence level, which implies that the suspension is distorting significantly.

We conclude that distortion of the Zollner-type wire suspension used in the WA seismometers is significant and must be taken into account when calculating the theoretical geometrical magnification. Thus the long-standing discrepancy in the static magnification values of 2800 given in the literature and 2080 ± 60 determined from the free period and tilt sensitivity is resolved.