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Coda

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Abstract—Observations and analysis of seismic scattering in the heterogeneous earth have grown from the initial observations of Aki in the 1960s into a well-developed subfield of seismology. The area presents many challenging and interesting problems for seismologists today and there are many areas of fruitful research. We focus on a small subset of areas of research within the general area of, ''coda study,'' that can be most directly tied to Kei's early work in this field: scattering coefficient, coda Q, coda normalization method, and the radiative transfer approach. These are the most useful tools in the interpretation of high-frequency seismograms. In each of these areas, Kei provided initial inspiration through insightful observation and well-thought out models for his observations. The results of ongoing work in these areas have provided insight into the complexity of wave propagation in the earth and have yielded new insights into the character of the earth's lithosphere. They have also provided reliable means to obtain practical information like relative site amplification factors and relative source radiation as a function of frequency.

Key words: Scattering, coda, seismology, heterogeneity, attenuation, Q.

Introduction

In 1969, Keiiti Aki first called attention to the continuous wavetrains in the tail portion of seismograms (AKI, 1969). Kei called these wavetrains ''coda waves'' and this term has been used since to describe the tail portion of regional seismograms. He observed that early portions of seismograms seem to be composed of waves that were propagating away from the source and decrease with amplitude with increasing propagation distance. Conversely, he observed that the coda had similar amplitude at all stations independent of epicentral distance and had similar spectral content among stations. He also noted that these waves were not simply correlated from station to station but rather appeared as an incoherent series of waves. Aki proposed that the appearance of coda waves was caused by the incoherent waves scattered from random heterogeneity in the earth's lithosphere. His original proposal was that these waves were surface waves. Characteristic of Kei, he argued that these waves,

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while they may seem complicated, were a tremendous resource because they provided a means to estimate source excitation characteristics (AKI, 1969). In that paper, Kei proposed to use coda spectra for the estimation of source spectra from seismograms in which the direct arrivals are clipped. Figure 1 shows traces of an earthquake recorded at two local stations in Japan. The *P* and *S* arrivals and *P* and *S* codas are labeled.

Following the idea of random heterogeneity influencing seismic observations, AKI (1973) investigated travel time and amplitude fluctuations at the seismic array in Montana (LASA), using predictions in CHERNOV's (1960) book on wave propagation in a random medium. In this paper, Kei argued that observed travel time and amplitude fluctuations could be explained by considering the effects on both of scattering from random heterogeneity.

Following on the observations of some of Kei's very early work (for example, AKI, 1956; AKI and TSUJIURA, 1959), and observations made by others (NIKOLAYEV and TREGUB, 1970; DAINTY *et al*., 1974; CAPON, 1974; NAKAMURA *et al*., 1970, Kei and Bernard Chouet wrote a paper that outlined their combined ideas regarding coda waves (AKI and CHOUET, 1975). They concluded that at low frequencies, less than about 10 Hz, coda waves are dominated by scattered surface waves but that higher frequency coda waves are comprised mostly of scattered body waves. They also outlined the following basic characteristics of coda waves at epicentral distances of less than about 100 km and at times greater than about twice the *S*-wave travel time: (1) the spectrum of the coda waves for one earthquake is the same at all recording

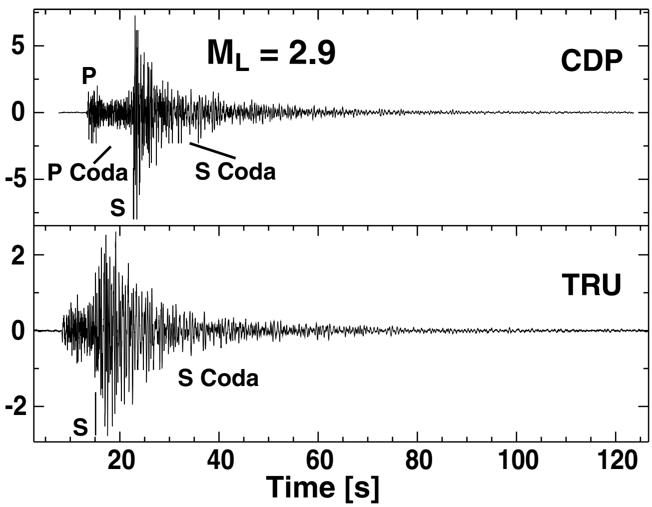

Figure 1
Traces recorded at two stations of an earthquake in Japan. Direct *P* and *S* arrivals are labeled. *P* and *S* codas are also labeled.

stations, (2) the coda length can be used as a reliable measure of earthquake magnitude (TSUMURA, 1967), (3) the power spectra of coda waves decays as a function of time in the same manner at all stations and for all events within a given region (RAUTIAN and KHALTURIN, 1978), (4) the temporal decay shape described in (3) is independent of earthquake magnitude for events with magnitude less than about 6, and (5) coda amplitude varies with local geology at a recording site. AKI and CHOUET (1975) introduced the parameter coda Q as a measure of the decay rate of the coda within a given frequency band, and they showed that this decay rate was independent of recording site and event location, provided that observations were made within approximately 100 km of the epicenter of an event. They also proposed to estimate the scattering power of random heterogeneity of the earth medium from coda excitation of local earthquakes.

Observations by TSUJIURA (1978) showed that the site amplification determined for coda waves agreed well with that determined from direct S waves but differed from that for P waves. This observation, combined with an apparent agreement between the coda Q and measurements of shear wave Q, led Kei to propose that coda waves are dominated by S waves and that attenuation in the earth is dominated by scattering (AKI, 1980).

Later work by Kei, his students, and collaborators was conducted to both refine observations of coda waves and improve our models for coda waves and/or high-frequency seismogram envelopes. Many of these studies are thoroughly discussed in a recent book (SATO and FEHLER, 1998), whose completion is a monument to Kei's influence on many scientists.

Since there are so many areas of study of coda waves, both observational and theoretical, we have chosen to limit our focus. We choose to discuss observational methods that can be most directly related to Kei's early observations and ideas, namely coda Q and the coda normalization method. We introduce some justification for the assertion by Kei in his 1969 paper that at a given time, the power spectrum of coda waves of a local earthquake is nearly independent of epicentral distance. Finally, we mention a more recent area of investigation, the use of the radiative transfer theory, which was introduced into seismology by one of Kei's students (WU, 1985). Later this approach was developed to correctly adopt the nonstationary process for the case of earthquake source radiation by Kei's students and collaborators. This approach has offered a way to synthesize the whole seismogram envelope and has provided a reliable method for estimating the amount of attenuation caused by scattering and intrinsic mechanisms.

Scattering Coefficient and Coda Q

The conceptual model of coda excitation was introduced by AKI and CHOUET (1975). Their single backscattering model is proposed to explain phenomenologically the coda envelopes of local earthquakes. They wrote the coda power as a convolution of the source, scattering and site amplification as

$$\left\langle \dot{u}_{ij}^{\mathrm{S\,Coda}}(t;f)^2 \right\rangle_{\mathrm{T}} = \frac{W_i^S(f) g_0(f) N_j^S(f)^2}{2\pi\beta_0^2 t^2} e^{-Q_C^{-1} 2\pi f t} \quad , \tag{1}$$

where $\dot{u}_{ij}^{\mathrm{S\,Coda}}(t;f)$ is the *S*-coda particle velocity at site *j* at center frequency *f*, $\langle\ldots\rangle_{\mathrm{T}}$ means average over some time window *T*, $W_i^S(f)$ is the *S*-wave source power for the i$^{\mathrm{th}}$ earthquake, $g_0(f)$ is the total scattering coefficient representing the scattering power per unit volume, $N_j^S(f)$ is the site amplification factor, and β_0 is the background *S*-wave velocity. They introduced coda *Q*, Q_C as a parameter to account for anelastic loss of energy from the wavefield. Later, JANNAUD *et al.* (1991) numerically confirmed the validity of relation (1).

Measurements of g_0 and Q_c are made by applying equation (1) to bandpass-filtered seismic data. Some investigators used an empirical relationship between radiated energy and local earthquake magnitude to estimate $W_i^S(f)$. Others used joint analysis of direct *S*-wave and *S*-coda envelopes. By first multiplying the filtered seismic trace data by *t* and then taking the logarithm and plotting it *vs*. lapse time *t*, Q_c can be determined from the resulting slope. Measurements are made using data over a given lapse time range that generally begins long after the *S*-wave onset at each station.

The total scattering coefficient $g_0(f)$ has been measured in many regions worldwide. Roughly, this parameter is the ratio of *S*-coda power to radiated *S*-wave source power. SATO and FEHLER (1998, Figure 3.10) compiled reported values of g_0, where they included reported total scattering coefficients estimated by assuming isotropic scattering. We find that total scattering coefficient g_0 for *S*-to-*S* wave scattering is of the order of 10^{-2} km^{-1} for frequencies 1–30 Hz and the scatter is a factor of two for individual measurements of g_0; however, regional differences related to tectonic activity have not yet been quantitatively clarified.

Observations of Q_c have been made by numerous authors using data from networks at many locations worldwide. Results confirm the conclusions of AKI and CHOUET (1975) that Q_c is relatively constant within a given region. Q_c varies with frequency, lapse time interval used in the observations, and tectonic region. It has generally been found to be lower in tectonically active regions and higher in tectonically stable regions. Changes in Q_c have also been suggested as a precursor to earthquakes although some have criticized precursory studies. In general, Q_C^{-1} is about 10^{-2} at 1 Hz and decreases to about 10^{-3} at 20 Hz. The frequency dependence within a region can be written as $Q_C^{-1} \propto f^{-n}$ for $f > 1$ Hz, where the power *n* ranges between 0.5 and 1. The variation from region to region is more than a factor of 10. Summaries of observations of Q_c can be found in SATO and FEHLER (1998) and SATO *et al.* (2002).

The relation between Q_c and intrinsic and scattering *Q* is not obvious. It is clear from reading papers of Aki that he struggled with this issue. He alternatively speculated that Q_c is the intrinsic loss (AKI and CHOUET, 1975) and equivalent to total attenuation which he said is dominated by scattering loss (AKI, 1980). FRANKEL and WENNERBERG (1987) developed the energy-flux model for coda waves that was based on the idea that energy is uniformly distributed within some region surrounding an earthquake at some lapse time after an earthquake and that Q_c is intrinsic loss. GUSEV (1995) demonstrated that coda decay is quantitatively well explained if the total scattering coefficient decreases with depth, when the leakage of scattered energy to the bottom cannot be discriminated from intrinsic loss. Later, MAYEDA *et al.* (1992) examined the relation among scattering Q and intrinsic Q determined using the radiative transfer theory and Q_c. They concluded that there is no simple relation between Q_c and scattering and intrinsic Q. It is necessary for us to develop an interpretation of coda Q that includes the depth dependence of the total scattering coefficient and intrinsic absorption in addition to velocity structure.

Coda Normalization Method

The coda waves provide a reliable way to isolate and quantify the seismic source radiation and receiver site amplification. It also allows the investigation of propagation effects. The fundamental empirical base of the coda normalization method is that there is a uniform distribution of coda energy in some volume surrounding the source at some lapse time. The key observation in support of this method is that coda envelopes have a common decay curve that is independent of the source-receiver distance (RAUTIAN and KHALTURIN, 1978; AKI and CHOUET, 1975). Coda amplitudes vary with source size and recording site amplification.

Site Factors

When the lapse time t_c is large enough, the relative coda amplitude at two different sites j and k should be the same except for the influence of the near-recording site amplification since the source factor is common:

$$N_j^S(f)/N_k^S(f) = \sqrt{\left\langle \dot{u}_{ij}^{S\,\mathrm{Coda}}(t_c;f)^2 \right\rangle_{\mathrm{T}} \Big/ \left\langle \dot{u}_{ik}^{S\,\mathrm{Coda}}(t_c;f)^2 \right\rangle_{\mathrm{T}}} \ . \quad (2)$$

TSUJIURA (1978) reported that the frequency dependence of site amplification factors estimated by the coda normalization method is more stable than those estimated from direct wave analysis. He also found that the amplification determined by averaging observations of coda amplification found for many earthquakes was similar to that obtained by averaging measurements of direct S waves. This conclusion was important to Aki's later assertion that coda waves are composed predominately of scattered S waves (AKI, 1980). Analyzing local earthquakes in northern California, PHILLIPS and AKI (1986) determined site amplifications for 150 stations. SU *et al.* (1992) extended the study of PHILLIPS and AKI (1986) and found that site amplification in central California is related to the geologic age of rocks at the receiver site. SU and AKI (1995) used coda waves to determine site amplifications of 158 stations in Central and Southern California and confirmed the relation between geologic age and site amplification found by SU *et al.* (1992). FEHLER *et al.* (1992) developed a map of site amplification factors for the Kanto–Tokai region of Japan. They showed that the site amplification factors at 6 Hz agreed well with local magnitude residuals, which were obtained from maximum amplitude measurements for the same stations.

Detailed comparisons of the relative site amplification factors determined using *S* waves and coda waves have led to the conclusion that there can be differences between the two, particularly in structurally complex areas (e.g., BONILLA *et al.*, 1997); however, other studies indicate that the differences are small (e.g., KATO *et al.*, 1995). However, the differences can be reduced with an appropriate choice of reference site or when the average response of multiple sites is used as a reference site.

Source Size

From equation (1), we ascertain relative source radiation as a function of frequency by dividing the coda amplitude of the seismogram recorded at one site *j* for a given earthquake *i* by the amplitude at the same site for a different earthquake *k* taken at the same absolute lapse time t_c at the same site *j*:

$$W_i^S(f)/W_k^S(f) = \left\langle \dot{u}_{ij}^{S\,\mathrm{Coda}}(t_c;f)^2 \right\rangle_{\mathrm{T}} \Big/ \left\langle \dot{u}_{kj}^{S\,\mathrm{Coda}}(t_c;f)^2 \right\rangle_{\mathrm{T}} \quad . \tag{3}$$

We know that coda wave excitation is mostly insensitive to radiation-pattern differences. Using seismic moments from two well-analyzed earthquakes in Alaska, BISWAS and AKI (1984) developed a coda-amplitude vs. seismic-moment scale. That relationship can be used to make a fast and reliable determination of the size of an earthquake using data from only one station. DEWBERRY and CROSSON (1995) performed a detailed analysis of the source spectrum of earthquakes in the U.S. Pacific Northwest using data from coda waves. HARTSE *et al.* (1995) determined source radiation as a function of frequency for earthquakes and nuclear explosions in the U.S.A.

Single Station Attenuation Measurements

The single station method for measuring attenuation was proposed by AKI (1980). By normalizing direct *S*-wave amplitude by *S*-coda amplitude, this method corrects for source size and site amplification, thus allowing data to be combined from many earthquakes to find a more stable estimate of attenuation. The direct S-wave particle velocity amplitude at station j at frequency f for local earthquake i is

$$\left|\dot{u}_{ij}^{S\ \mathrm{Direct}}(f)\right| \propto \frac{1}{r_{ij}}\sqrt{W_i^S(f)}N_j^S(f)e^{-Q_S^{-1}\pi f\ r_{ij}/\beta_0}\ , \tag{4}$$

where r_{ij} is the source-receiver distance, β_0 is S-wave velocity, and Q_S is the Q of direct S waves. Taking the logarithm of the ratio of the product of hypocentral distance and the direct S-wave amplitude to the averaged coda amplitude given by equation (1), the common site amplification and source terms cancel, we get

$$\ln\frac{r_{ij}\left|\dot{u}_{ij}^{S\ \mathrm{Direct}}(f)\right|}{\sqrt{\left\langle \dot{u}_{ij}^{S\ \mathrm{Coda}}(t_c;f)^2\right\rangle_{\mathrm{T}}}} = -\left(Q_S^{-1}(f)\pi f/\beta_0\right)r_{ij} + \mathrm{Const}\ . \tag{5}$$

We smooth out the radiation pattern differences when the measurements are made over a large enough number of earthquakes. Plotting the left-hand side against hypocentral distance, the gradient gives the attenuation per travel distance. AKI (1980) first applied this method to seismograms of local earthquakes recorded in Kanto, Japan. His work stimulated measurements of attenuation in many areas of the world during the 1980s. Later, YOSHIMOTO *et al.* (1993) extended the conventional coda-normalization method to measure Q_P^{-1}.

Qualitative Discussion of the Coda Normalization Method

The foundation of the coda normalization method was stated near the beginning of AKI (1969) where he stated that "the power spectrum of coda waves at a given time measured from the earthquake origin time appears to be nearly independent of the epicentral distance." This observation is critical to the success of the coda normalization method. Stated another way; after some time, if energy is uniformly distributed within some region surrounding the source, all stations within that region will have trace amplitude that is proportional to the size of the earthquake. Variations in amplitude observed at differing stations are caused by local site amplification effects. These statements are implicit in equation (1).

The uniform distribution of coda energy within some region surrounding the source for lapse times greater than some lapse time is predicted by the single isotropic scattering model (see for example Figure 3.6 of SATO and FEHLER, 1998). Results calculated using the radiative transfer equation for models that include multiple scattering, the effects of nonisotropic scattering, and nonisotropic source radiation also indicate that energy is uniformly distributed within a medium after sufficient time has passed from the initial source excitation time (see for example Figure 7.24 of SATO and FEHLER, 1998).

Uniform distribution of coda energy was also observed in results of numerical simulations by FRANKEL and CLAYTON (1986). To examine this in more detail, we show results of 2-D numerical simulations of scalar wave propagation in a medium that is characterized by a von Kármán autocorrelation function with $\kappa = 0.1$. A medium with a von Kármán autocorrelation function is extremely rich in short wavelength heterogeneity. The power spectrum density (PSDF) of the medium heterogeneity is shown in Figure 2. Numerical modeling for a medium with a Gaussian autocorrelation function was discussed in FEHLER *et al.* (2000). They discussed a case in which forward scattering dominates because the dominant wavelength of the source excitation (2 km) was much less than the dominant wavelength of the medium heterogeneity (5 km). In the simulations of FEHLER *et al.* (2000) no coda are evident in the vicinity of the source. In addition, after the initial wave packet had passed a given observation distance, there is no coda at that distance.

Our numerical modeling was conducted using a finite-difference simulation of the 2-D scalar wave equation in the same manner as described in FEHLER *et al.* (2000). We used a medium with a correlation length of 5 km and a RMS fractional fluctuation in the velocity of 5%. We note that the wavenumber corresponding to 2 Hz is about 3.1/km, which is in the power-law range of the von Kármán type PSDF with $\kappa = 0.1$ for the background velocity of 4 km/s. Due to the presence of small-

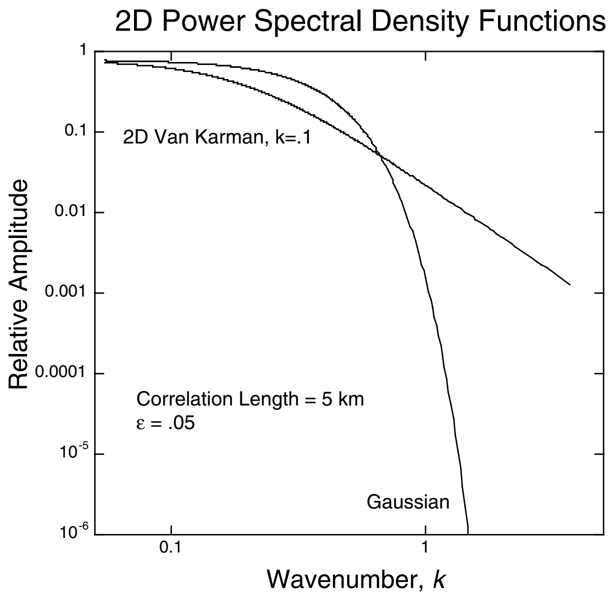

Figure 2
PSDF for random media with 2-D Gaussian and von Kármán autocorrelation functions. Correlation length of both media is 5 km and RMS velocity fluctuation is 5%. Note that the medium with von Kármán autocorrelation function has more short wavelength inhomogeneity; e.g., more power in large wavenumbers, than the medium characterized by a Gaussian autocorrelation function.

scale heterogeneity, coda waves are formed by backscattering near the source. Figure 3 displays ensemble-average RMS envelopes at distances ranging from 25 to 200 km from the source, constructed from 50 numerical simulations. The procedure for calculating the ensemble average envelopes is described in FEHLER *et al.* (2000). Contrary to the results obtained from modeling propagation in a Gaussian random medium where correlation distance is larger than the dominant wavelength, the envelopes here show well-developed coda with a long tail at short distances. Note also that at large lapse times, e.g., times much later than the direct arrival time at a given distance, the coda amplitude reaches a level that is independent of observation distance. The constant level of coda amplitude for the case of random media possessing spectra rich in short-wavelength components is in agreement with the phenomenological observation that the coda normalization method is reliable.

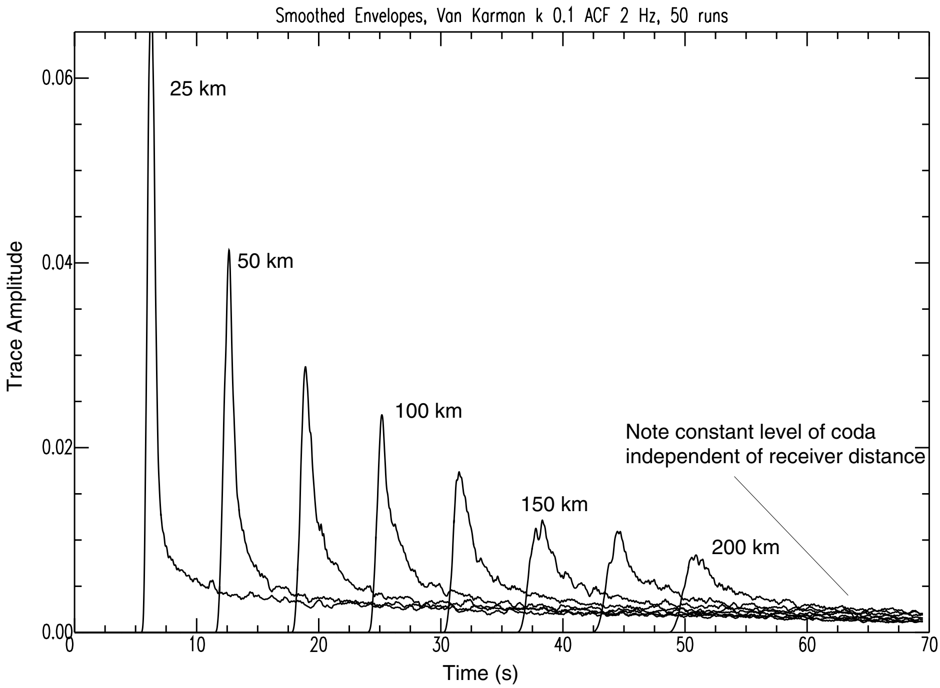

Figure 3
Ensemble-average RMS envelopes calculated for media with von Kármán autocorrelation function whose PSDF is shown in Figure 2. Since source was a 2 Hz. Ricker wavelet, envelopes should be considered to be envelopes of seismic waveforms that have been band-passed in a frequency range from about 1–4 Hz. Note coda of envelopes at all distances whose amplitude at large lapse-time is nearly the same, independent of distance from the source.

Power-law Spectra

Analyzing the frequency dependence of *S*-wave attenuation and scattering coefficient revealed from coda excitation of local earthquakes, WU and AKI (1985) found that inhomogeneities in the lithosphere have power-law characteristics. This means that the spectral content of random inhomogeneities of the earth is rich in short wavelength components compared with the Gaussian spectra. Direct evidence was found from the study of well log data. SATO (1979) reported that the autocorrelation of a borehole acoustic velocity log in crystalline rock in Kanto, Japan is not Gaussian but exponential-like, peaking sharply at lag distance zero. Analyzing sonic log data of the KTB deep-borehole in Germany, WU *et al.* (1994) reported that the spectrum of heterogeneity obeys a power-law for a wide wavelength range. LEARY and ABERCROMBIE (1994) also reported that the PSDF of sonic velocity log at the Cajon Pass borehole in California obeys a power-law.

Analyzing coda of a microearthquake recorded by a borehole seismometer located at 2.5 km depth in the Cajon Pass hole, LEARY and ABERCROMBIE (1994) found that the spectral ratio of coda spectra to *S*-wave source spectra is proportional to the spectra of acoustic reflectivity log data for a broad frequency range from 10 to 200 Hz. As discussed in the previous simulation, the uniform distribution of coda energy might be related with such spectral content. It is necessary for us to further study seismic wave scattering through random media with such power-law spectra.

Radiative Transfer Theory

Aki's initial models explaining seismic coda waves were based on the use of a single scattering approximation. The single scattering assumption worked well to explain seismic observations even though there were concerns raised about the importance of multiple scattering (see e.g., AKI and CHOUET, 1995). Several attempts were made to investigate the importance of multiple scattering as observations were made that indicated that it might be an important factor. The radiative transfer theory has been an attractive theory because it allows relatively tractable calculations of effects of multiple scattering. The radiative transfer theory was first introduced into seismology by WU (1985) although it had been used in other areas of physics for several decades (CHANDRASEKHAR, 1960). The formulation for the nonstationary state was done for the 2-D case by SHANG and GAO (1988). ZENG *et al.* (1991) beautifully formulated the non-stationary multiple scattering process in 3-D. The diffusion model, which was used for the study of lunar seismograms (DAINTY *et al.*, 1974), is considered as an extreme limit of the multiple scattering process.

Here, we introduce the equation that governs radiative transfer for media with multiple isotropic scattering. We imagine a 3-D scattering medium with the background propagation velocity V_0, in which point-like isotropic scatterers of cross section σ_0 are randomly and homogeneously distributed with number density n, where $g_0 \equiv n\,\sigma_0$ is the total scattering coefficient characterizing the scattering power per unit volume. We assume an impulsive radiation of energy W at time zero from a source located at the origin. The energy density due to the propagation of coherent waves from the source is $We^{-(V_0 g_0+\eta)t} \times \delta(t - r/V_0)/4\pi V_0 r^2$. We include the exponential decay term $V_0\,g_0$ to account for scattering loss and η for intrinsic attenuation per time. Generation of scattered energy per unit time from a unit volume at the last scattering point $(\mathbf{x}', t')$ is a product of g_0, V_0, and energy density $E(\mathbf{x}', t')$. Including geometrical spreading and the time lag due to propagation, we derive the energy-flux density at the receiver at ($\mathbf{x}$, t) due to scattered waves from a unit volume. Integrating the scattered energy-flux density over the entire 3-D space and dividing by V_0, we derive the total contribution of scattered energy density. Adding the energy density of direct propagation of coherent waves from the source to that of the scattered waves, we obtain

$$E(\mathbf{x}, t) = W\ G_E(\mathbf{x}, t) + g_0 V_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_E(\mathbf{x} - \mathbf{x}', t - t')E(\mathbf{x}', t')\,d\mathbf{x}'dt' \quad (6)$$

where the direct propagation of coherent wave energy is given by

$$G_E(\mathbf{x}, t) = \frac{1}{4\pi V_0 r^2} H(t)\delta\left(t - \frac{r}{V_0}\right) e^{-(g_0 V_0+\eta)t} \quad . \quad (7)$$

Solutions of equations (6) with (7) yield information about the distribution of energy in space and time. The solution in the time domain gives the mean-square envelope of the entire wavefield. The calculations are conducted in terms of energy since the wavefield is assumed to be composed of incoherently scattered waves. Solutions of the radiative transfer equation which have forms similar to equation (6) for various assumptions pertaining to source and media are a fruitful frontier area of research. For example, the effects of multiple scattering on the polarization of S waves have recently been studied (BAL and MOSCOSO, 2000) and Monte Carlo simulations have been used to investigate radiative transfer in an elastic medium (MARGERIN *et al.*, 2000).

The radiative transfer approach forms the basis of a viable analysis method for making independent estimates of the amount of scattering that is caused by intrinsic and scattering mechanisms. An initial approach for making these measurements was proposed by WU and AKI (1988). WU and AKI (1988) used their stationary-state solution to analyze data. However, application of this solution required the measurement of the total energy in a seismogram, which may be underestimated if significant portions of energy are buried in noise in later portions of the seismogram. The method was later modified with the introduction of the Multiple Lapse Time Window analysis Method (FEHLER *et al.,* 1992; MAYEDA *et al.,* 1992), which does not require the total energy in the seismograms to be measured. Results obtained using the radiative transfer theory show that at frequencies near 1 Hz, scattering is the dominant mechanism of attenuation. However, as frequency increases to 10 Hz, scattering looses importance as an attenuation mechanism and intrinsic (nonelastic) attenuation mechanisms dominate (Fig. 7.12, SATO and FEHLER, 1998). These results provide useful information for developing models of seismic attenuation and scattering.

Conclusions

Kei Aki once remarked that of all the work he had done in seismology, he was most proud of his work on coda waves because if he had not done the work, it is likely that no one else would have done so. With his remarkable insight, beginning with his observations in the 1950s, Kei has provided seismologists with a wealth of tools and ideas regarding wave propagation in the heterogeneous earth. He has inspired many of his students and other investigators to work on related topics. To his credit, many of the ideas and observations he made early in his career using very little data that were poorly recorded by today's standards are still viable today. Some of these observations still provide challenges to us today as we attempt to formalize the physics that explains his observations.

Coda waves have proved to be a valuable tool. With no formal theory to explain their formation, the coda normalization method is a valuable tool for making quantitative measurements of source radiation and site amplification effects. Enhancements to allow the measurement of seismic attenuation using a single station method as developed by AKI (1980) or using the radiative transfer theory have provided us with results that would not be achievable by other methods.

The number of researchers who are building careers out of using various aspects of the ideas initiated by Keiiti Aki for dealing with coda waves and wave propagation in random media is enormous. Each has added to our body of knowledge relating to the earth and each has gained enormously from the inspiration of Kei Aki.

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