Pure appl. geophys. 160 (2003) 579–602
0033 – 4553/03/040579 – 24

Pure and Applied Geophysics

Spontaneous Complex Earthquake Rupture Propagation

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Abstract — The historical development of spontaneous rupture propagation, starting from the landmark paper of Griffith in 1920, through to the late 1980s is traced, with particular emphasis on the work carried out at MIT in the 1970s by K. Aki and his co-workers. Numerical applications of Kostrov's method for planar shear cracks were developed by Hamano, Das and Aki. Simultaneously at MIT, Madariaga considered the radiated field of a dynamic shear crack. The further development of these ideas, for example, three-dimensional spontaneous planar faulting models, continued through the 1980s. Major insight into the maximum possible rupture speeds for earthquakes developed, with the acceptance of the theoretical possibility of supersonic rupture speeds for faults with cohesion and friction, the theoretical developments spurring the search for such observations for earthquake ruptures. Possible mechanisms by which faults stop were elucidated. It was shown that a propagating rupture can jump over barriers for cracks with a cohesive zone at its tip. Complex faulting models, namely the barrier and asperity models, and their associated radiated field developed. In the late 1980s, it was shown that "dynamic" or transient asperities can develop during the complex rupturing process. Even seemingly relatively simple physical situations, can lead to such complex rupturing processes that the usual idea of "rupture velocity" needs to be abandoned in those cases. Some of the work initiated by Aki and his co-workers, such as the details of the transition from sub-Rayleigh to super-shear speeds in inplane shear mode, and the behavior of the cohesive zone size as the crack extends, still remains the subject of research today.

Key words: Earthquake rupture, complex faulting, spontaneous rupture propagation.

Introduction

KOTO's (1893) study of the 1891 Mino-Owari earthquake finally confirmed the faulting origin of earthquakes, though a few seismologists still continued this debate into the 1960s! Before then, cause and effect were confused. Even a scientist as great as Darwin, in his description of the 1835 Chilean earthquake, wrote: "The most remarkable effect of this earthquake was the permanent elevation of the land; it would probably be far more correct to speak of it as the cause" (DARWIN, 1889). In the history of seismology, "who first proposed it" (that earthquakes are due to faulting) "is not definitely known" (HOWELL, 1990). Clearly, REID's (1910) brilliant study of the 1906 San Fransisco earthquake soon after Koto's paper helped in its acceptance. But though it was understood by Reid that shallow earthquakes were

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due to rupture of the earth's crust in response to tectonic stresses, the physical basis for analyzing this phenomenon, namely, fracture mechanics, did not yet exist. It was only in 1920 that Griffith initiated the study of the mechanics of fracturing, and the subject really developed vigorously during and after the Second World War. GRIFFITH (1920) understood that for a pre-existing flaw in a material to extend, the energy required to create new crack surface (the fracture energy) can be at most equal to the strain energy available in the body. This criterion, now known as the "Griffith fracture criterion" is a global criterion. In 1957, Irwin first introduced the idea of stress intensity factor k at the crack tip, the stress at the crack tip being given by $k/(\sqrt{r})$, where r is the distance from the crack tip outside the crack, together with higher order terms in r, which can be neglected as one approaches the crack tip (that is, as $r \to 0$). Using this, IRWIN (1957, 1958, 1969) developed a local criterion which states that the crack tip extends when the stress-intensity factor exceeds some critical value, called the fracture toughness of the material. These ideas were applied at first to quasi-static rupture of tension cracks.

The static solution for displacement on a inplane shear crack was written down by STARR (1928). ESHELBY (1957) wrote down the closed form solution for static elliptical cracks, both in tensile and in shear modes. These solutions indicated that for constant stress drop on a simple earthquake fault, the displacement is variable, decreasing from zero at the edges to a maximum at the center. KOSTROV and DAS (1984) evaluated and plotted the stresses around circular and elliptical faults using this latter solution.

Development of Methods of Solution for the Dynamic Problem

In the 1960s, Kostrov pioneered the application of ideas developed in fracture mechanics to the study of shear fracture and hence set up the basis for analyzing earthquake ruptures. In KOSTROV (1964) he published the analytical solution for a self-similar shear crack extending at a prescribed velocity and showed that for a constant stress drop on the crack, the fault slip velocity varies at the crack edge as the inverse of the square root of the distance of any point on the crack to the crack edge. His solution also shows that the displacement in time at each point of the crack increases from zero to its final constant slope value, and that the time to reach this value increases as one moves away from the point of rupture initiation. This implies that the "rise time" of the source time function varies over the fault. In KOSTROV (1966), he considered the propagation of the semi-infinite antiplane shear crack which suddenly appears and starts extending at a prescribed (but not necessarily constant) velocity without stopping. He wrote down the complete closed form solution for this mixed boundary value problem, in which the stress changes are assumed known within the crack (the stress drop) and the displacements are known outside the crack (the slip is zero there). This solution gives the displacement on such a crack for any known stress drop distribution on it, as well as the stresses in the causal region outside the crack. This method has since been called the "Green function method" by FREUND (1990), and had been widely used in potential theory and in fluid mechanics studies of supersonic flow around aerofoil wings (developed by EVVARD (1950) and described in his textbook by WARD (1955)). Note that the equations for a crack tip moving through a solid is identical to those for fluid flowing past a solid object! BURRIDEGE and WILLIS (1969) developed the dynamic solution for a self-similar elliptical crack.

Kostrov's 1966 method formed the basis of the numerical boundary-integral equation (BIE) method later developed by HAMANO (1974) and DAS and AKI (1977a) for 2-D problems, and by DAS (1980) and DAS and KOSTROV (1987) for 3-D problems. In this method, the problem reduces to calculations of quantities on the fault surface only. The problem formulation in 3-D is briefly described below, mainly for completeness, and for definition of quantities to be used later in this paper. The earthquake source is modeled as a propagating plane shear crack in an infinite medium (Fig. 1) which is homogeneous and linearly elastic everywhere off the crack plane, the latter considered to have infinitesimal thickness. (Remember that earthquakes cannot occur in a medium that is truly homogeneous everywhere.) As the fault propagates on the planar surface F: $X_3 = 0$, waves are radiated out in three spatial dimensions. Initially, the infinite body is under a uniform state of stress σ_{ij}^0. The initial stress on the fault plane $X_3 = 0$ can be separated into the normal stress σ_{33}^0 and a shear stress $\sigma_{13}^0 = \sigma^0$, say. The component σ_{23}^0 can be taken as zero by taking the coordinate axis X_1 in the direction of the maximum initial shear (without loss of generality). The initial shear stress is increased sufficiently to initiate a fault at the origin, which then propagates on the $X_3 = 0$ plane. The normal stress σ_{33}^0 over the fault plane remains constant throughout the rupture process, for a planar fault. Let us take the origin of time $t = 0$ as the time when the fault initiates and starts

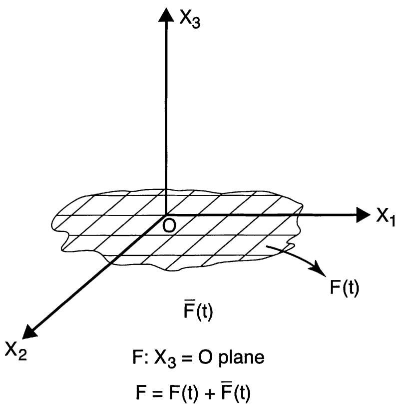

Figure 1
The geometry of the fault.

extending. We study the case when the fault propagation speed is rapid enough to generate elastic waves. The fault edges may move at some pre-assigned speed or the position of the fault edge may be found as a function of time, using some fracture criterion. (The latter is called "spontaneous" propagation in seismology.) Let us consider the former case only for the moment (that is, the fault propagation speed in all directions on the fault plane is known). As the fault propagates, there is relative motion between the two faces of the fault, that is between the regions $X_3 < 0$ and $X_3 > 0$, and a displacement discontinuity appears across the broken region of the fault plane. This discontinuity is a function only of the coordinates X_1 and X_2 and time t. The shear stress on the fault surface is zero if there is complete stress release; or, it can be equal to the the frictional stress σ on the fault faces, given by $\sigma = \mu\sigma_{33}^0$, where μ is the coefficient of friction. μ may be taken constant or a function of space, time, and any other desired parameter. Let the incremental stresses due to the displacement $\mathbf{u}$ from its initial configuration be τ_{ij}, so that $\sigma_{ij} = \sigma_{ij}^0 + \tau_{ij}$, that is, τ_{ij} is the stress change due to the motion, and all motions depend only on these stress changes on the fault. Exploiting the symmetries in the problem for planar shear cracks, the solution can be shown to be antisymmetric in X_3 that is, the displacement components u_1, u_2 and traction perturbation τ_{33} are odd in X_3 while u_3, τ_{13} and τ_{23} are even in X_3 (DAS and AKI, 1977a). Hence, it is sufficient to solve the problem for the upper half-space $X_3 \geq 0$. Further, from the continuity of tractions across $X_3 = 0$, it follows that τ_{33} vanishes everywhere on $X_3 = 0$. Then, the required representation relation is obtained as

$$u_k(\mathbf{X}, t) = \int_{-\infty}^{\infty} dt' \iint_F G_{ki}(\mathbf{X} - \mathbf{X}', t - t')\tau_{k3}(\mathbf{X}', t')dS \tag{1}$$

where $\mathbf{X}$ and $\mathbf{X}'$ are two-dimensional vectors on F, u_k is the component of displacement in the k direction, G_{ki} is the displacement response of the medium in the k direction at $(\mathbf{X}, t)$ due to an impulse acting in the i direction at $(\mathbf{X}', t')$, $k = 1, 2, 3$, and F is the causal portion of the fault plane $X_3 = 0$, that is, the cone of dependence given by

$$v_P^2(t - t')^2 - (X_1 - X_1')^2 - (X_2 - X_2')^2 \geq 0, \quad t \geq t' \geq 0 \tag{2}$$

where v_P is the compressional wave speed of the medium. The required components of the Green functions G are the solution to Lamb's problem and can be expressed in terms of elementary functions. The analytical expressions for G_{ki} for the two- and three-dimensional problems are given in Appendix I of KOSTROV and DAS (1988). The kernel G possesses only weak singularities and can be directly discretized for numerical computation.

This mixed boundary value problem is solved numerically by discretizing the above equation. However, the stress changes τ_{k3} are known only on the broken part of the fault plane (the stress drop) but are unknown in the unbroken but causal portion, so these have to be determined before the integrations can be carried out in equation (1). This is done by using the fact that the slip is zero on the unbroken part so that the lhs of (1) is zero there for $k = 1, 2$. The solution then proceeds by a time-marching scheme. The region of integration in 3-D, the intersections of the cones of dependence and influence, is shown in Figure 2 (DAS, 1980). Note that even though the problem was solved by DAS and AKI (1977a) in 2-D only, the full set of equations for the 3-D problem were written down by DAS (1976) [in fact, it follows straightforwardly from BURRIDGE (1969)] and the required Green functions were already available, having been written down by CHAO (1960) for a Poisson solid, (the expressions for a general solid were given by RICHARDS (1979)), so that the development of the 3-D problem later by DAS (1980) followed naturally.

The normal component of displacement u_3 is non-zero during the dynamic process, though of course there is no discontinuity in this component across the fault for the shear crack problem. This property had already been used earlier by AKI (1968) to study the near-field transverse component of 1966 Parkfield earthquake, the only near-field seismogram of that earthquake that was recorded.

In 1969, Burridge had started working on the problem of dynamic crack propagation, first in 2-D, later extending his method, also a numerical boundary-integral method, to some simple (namely, the acoustic) 3-D problem (BURRIDGE and MOON, 1981). DAS and KOSTROV (1987) have discussed this form in detail, and shown that these two forms of writing the BIE are mutually inverse integral transforms of one another. In this second form of the BIE, the stress on the fault is written as a convolution between a kernel and the fault displacement. It has the advantage that the integrations extend only over the slipping portion of the fault. It

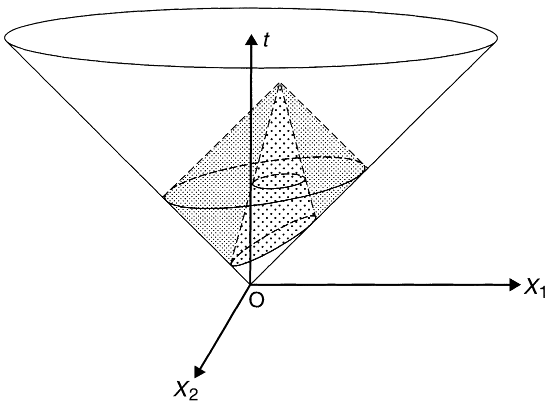

Figure 2
Volume of integration in BIE method of DAS (1980). The cone with vertex pointing down is the "cone of influence," and the cone with its vertex pointing up is the "cone of dependence." The inner cone (stippled) is the region where the Green function vanishes for this problem, so that the actual volume of integration for this problem formulation is the grey area.

has the disadvantage that the singularities in the kernel are strong and the kernel cannot be discretized as simply as in the previous form of the BIE. Burridge (1969), mentioned above, did this. Das and Kostrov (1987) determined another numerical form of the kernel. Most recently, Madariaga and Cochard (1992) and Cochard and Madariaga (1994) have used this form of the BIE in their work, discretizing the kernel directly. This latter form of the BIE is particularly suited to "interior" crack problems such as an expanding fault, whereas that given in equation (1) is most efficient for "exterior" crack problems such as the rupture of an asperity on an infinite fault, where the zone of the unknown stress changes decreases with time.

Simultaneously, programs were developed to study the crack problem using finite-difference and finite-element methods, both in fracture mechanics for tension cracks in 2-D, and by seismologists for shear cracks, both in 2-D and 3-D. For the tension crack, the reader is referred to the very comprehensive bibliography given by Freund (1990). For the shear problem, the 2-D work was carried out by Andrews (1976a,b; 1985) and by Archuleta (1976) and later by Day (1982a,b) in 3-D.

Kostrov's *chef d'oeuvre* was probably his 1975 paper (Kostrov, 1975) in which he wrote down closed form expressions for all three modes of semi-infinite and finite crack propagation for cracks with variable velocity, as long as this velocity did not exceed the Rayleigh wave speed. This paper is a landmark in the development of applied mathematical methods in fracture mechanics, as John Willis recently reminded us at the memorial meeting in honor of Kostrov, held at the EGS meeting in 1999 at The Hague. But probably due to its cumbersome nature, this solution has not been used widely by seismologists, though it is used by fracture mechanicists.

Spontaneous Propagation of Cracks

In his 1966 study, Kostrov also considered the problem of spontaneous propagation of a semi-infinite antiplane shear crack that suddenly appears and starts extending. Using a dynamic form of Griffith's criterion, he showed that if the material through which the crack is extending has constant fracture energy γ, the crack goes through a stage of accelerating from zero to its final speed and then continues to propagate at this speed. For the antiplane crack, this terminal speed was the shear-wave speed of the medium, but for stronger materials (i.e., with higher γ), the time to reach this speed was longer. Kostrov's 1966 paper was a very short paper, demonstrating that the length of a paper is unrelated to its impact on a field! In the introduction of his book "Cracks and Fracture," Broberg (1990) singles out the work of Kostrov and Freund in dynamic fracture mechanics by saying "In the dynamic field, the significant and pioneering contributions of B. V. Kostrov and L. B. Freund deserve particular mention."

During his studies in the 1960s, Kostrov found that despite the sophisticated developments in fracture mechanics, for seismologists to apply the ideas to earthquake rupture was not straightforward. The reasons for this were twofold. First, engineering structures are in tensional stress regimes and all the theories were relevant to tensional fracture. In tensile cracks, the two faces of the crack are not in contact and hence there is no friction between them. For shear cracks, of course, this is not the case. In particular, when considering the total energy balance budget in a problem, the friction of the shear -fault surfaces becomes very large. Later, FREUND (1979) showed by considering the energy budget of dynamically propagating cracks that as a shear fault becomes larger and larger, the frictional term becomes more and more dominant over the term representing the energy needed to create new fracture surface. Studies of antiplane shear in fracture mechanics were considered in a similar way to the tension crack problem, and friction was always neglected. Secondly, at that time, most engineering studies related to quasi-static fracture, but earthquake ruptures are a dynamic phenomenon. KOSTROV and NIKITIN (1970) even had to redefine the idea of fracture by what they termed the "model of fracture." Without going into details here, we simply refer to his original paper as well as to a brief description in KOSTROV and DAS (1988). The crack tip energy flux for dynamic fracture was proposed by ATKINSON and ESHELBY (1968) simply by taking that for quasi-static fracture and guessing the result for dynamic propagation. KOSTROV and NIKITIN (1970) confirmed these expressions for dynamic fracture working directly from the field equations. KOSTROV and NIKITIN (1970) also extended the idea of the path-independent J-integral (ESHELBY, 1956; RICE, 1968) for quasi-static fracturing to dynamic fracturing (the J-integral can be obtained from their expression simply by setting the rupture speed to zero).

BURRIDGE (1973) demonstrated that for cohesionless cracks with friction, the maximum rupture speed could reach the compressional wave speed of the medium. His results were not taken seriously either in fracture mechanics or in seismology due to the cohesionless nature of the crack.

In parallel with developments of the theories which would lead to the study of spontaneous fault rupture, some important milestones occurred around this time in the study of the earthquake source. The body force equivalent in terms of the double-couple was developed by BURRIDGE and KNOPOFF (1964), and the scalar seismic moment was defined by AKI (1966). RANDALL (1971) understood that the seismic moment was actually a tensor. BRUNE (1970) wrote a simple relation between earthquake stress drops and fault radius, for a circular fault, and since this was a simple formula, it became very widely used, sometimes for faults that were far from being equidimensional.

The Cohesive Zone Model

In linear elastic brittle fracture, the transition from broken to unbroken material occurs over an infinitesimally small region. This sharp transition leads to infinite stresses at the crack tip in mathematical considerations. But since this cannot exist in reality, there is, in fact, a region between these two states where the material may be partially broken. Such a model of fracture is termed imperfectly or non-ideally brittle, and this intermediate region is called the "cohesive zone." BARENBLATT (1959) introduced the idea that the bonding force between atoms that end up on opposite faces of the crack after total separation, is proportional to the separation distance between them while the atoms are still in the transition or cohesive zone. Almost simultaneously, LEONOV and PANASYUK (1959) in the then USSR (the original paper was written in Ukrainian and hence is not easily accessible to most readers!) and DUGDALE (1960) in the United States developed the idea of the process zone further.

From 1972 to 1973, Ida, working beside Aki at MIT, developed a cohesive zone model in which the cohesive zone stress depends on the amount of relative slip between the two faces of the crack for a shear crack (IDA, 1972, 1973). PALMER and RICE (1973) introduced a similar idea to study the stability of a slope under gravitational sliding. In these models, now termed "slip-weakening" models, the stress just outside the crack is shown in Figure 3, and the work done at the crack tip, or the fracture energy, is the shaded area. FREUND (1990) explores such models more fully in his Chapters 5 and 6. ANDREWS (1976a) showed that for such a model the size of the cohesive zone decreases as the crack length increases, whereas for a "strain-weakening model," it remains constant. Based on laboratory results, OHNAKA (1996) stated that "the size of the breakdown zone is almost constant in the zone of dynamic, fast-speed rupture propagation." This is also seen from Figure 26 of OHNAKA and SHEN (1999).

BRACE and WALSH (1962) measured fracture energy in quartz in the laboratory under shear stress. IDA (1972) estimated the fracture energy for earthquakes from his

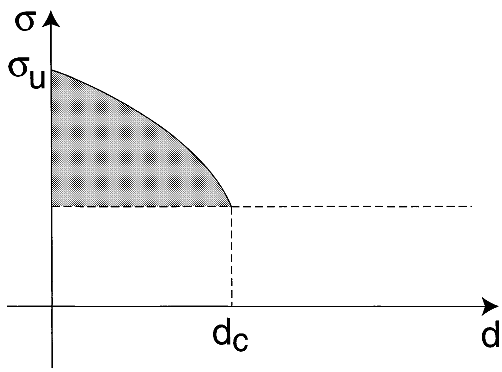

Figure 3
Cohesive zone model, showing the stress σ at the crack edge plotted against the slip weakening distance d. The critical distance is d_c and the shaded area gives the fracture energy.

cohesive zone model, and found that it is several orders of magnitude greater than that obtained in laboratory experiments. All further estimations of fracture energy (DAS, 1976; AKI, 1979) found similar results.

The critical stress level fracture criterion. The ideas of Griffith, Irwin, Barenblatt, etc. all lead to seemingly different fracture criteria. KOSTROV (1966) showed that the Griffith and Barenblatt criteria were equivalent from energy arguments and WILLIS (1967) proved their equivalence by a direct stress analysis. For numerical applications however, these criteria are difficult to use. So first HAMANO (1974) and then DAS and AKI (1977a) introduced the "critical stress level fracture criterion" in which a grid ahead of the crack tip is allowed to break when the stress in that grid exceeds some critical stress level related to the resistance of the material to fracture. Hamano's preliminary results were never published, except as an AGU abstract. Work on this problem was continued by Das and Aki between 1974–1976. Hamano had started developing the 2-D numerical form of KOSTROV's (1966) Green function method for a semi-infinite antiplane shear-crack solution, and extended it to 2-D finite cracks for all three crack modes. He also implemented the critical stress level fracture criterion, but did not show its connection with the Irwin criterion, which was done later by DAS (1976) and DAS and AKI (1977a). They related this criterion to a discrete form of the Irwin criterion, but of course the relation is grid-size dependent. VIRIEUX and MADARIAGA (1982) studied this criterion further and by comparing the analytical and grid-size-dependent numerical solutions for the antiplane shear crack determined the range of normalized critical stress levels for which this criterion gives the same result as the analytical solution using the Irwin criterion.

The critical stress level criterion becomes nonproblematic from the point of view of grid size dependence if we consider the material to be imperfectly brittle and the stress at the crack tip to be nonsingular. Then for small enough grid sizes (but not so small as to be impracticable for computations) the average stress near the crack edge varies smoothly and becomes independent of the grid size. In seismological applications, no attempt has ever been made to relate this discretized resistance to fracture at the crack edge to actual laboratory measurements. This is partly because it is not yet technologically possible to conduct dynamic fracture experiments on large rock specimens at the temperatures and pressures that exist at depths in the Earth where earthquakes actually occur. So the numerical results have been discussed in the context of fracture on "relatively strong" or "relatively weak" faults or interfaces. The term "relative" is discussed next. For this, we define the dimensionless quantity $S = (\sigma_u - \sigma_0)/\sigma_e$, where σ_0 is the initial stress level, σ_u is the critical stress level ahead of the crack tip required for fracture, and σ_e is the stress drop on the fault. Then larger S implies "relatively" stronger material, that is, relative to the stress drop.

Relation between Different Fracture Criterion

DAS (1976) and DAS and AKI (1977a) extended Kostrov's analysis for the Griffith criterion to the Irwin criterion and compared both results with that for the critical stress level fracture criteria for a semi-infinite antiplane shear crack that suddenly appears and extends spontaneously. Using Griffith's criterion, the crack tip position x as a function of time t is

$$x = \beta t + \beta t_c(\pi/2 - 1 - 2\ \arctan(t/t_c))\ ,$$

where t_c is the time of onset of fracture (KOSTROV, 1966). Using Irwins's criterion, DAS and AKI (1977a) showed that the crack tip position in time is

$$x = \beta(t - t_c) - \beta t_c\ \log(t/t_c)\ .$$

Figure 4 shows the crack tip position as a function of time for this problem for the analytical forms of the Irwin and Griffith criteria and for the numerical criterion. It shows that for the same t_c, the Griffith locus always lies above the Irwin locus, i.e., the Griffith crack accelerates faster to the terminal velocity than the Irwin crack. For the numerical case, by finding the value of the (grid-size dependent) S for which a crack starts propagating at the same time as those for the Griffith and Irwin criteria, namely with the same t_c, it was shown that the critical stress level criterion could indeed be considered a numerical analog of the Irwin criterion. This criterion has since been used in numerical applications in seismology.

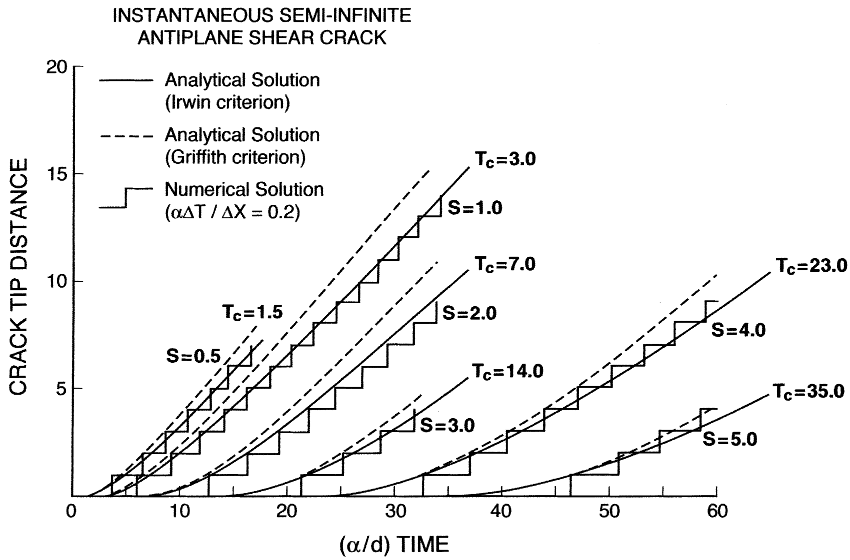

Figure 4
Comparison of the crack tip positions for the Griffith, the Irwin and the critical stress level fracture criteria (after DAS and AKI, 1977a).

Maximum Permissible Rupture Speeds for Shear Faults

BARENBLATT and CHEREPANOV (1960) and BROBERG (1960) showed that the maximum speed for tension cracks in perfectly brittle materials (i.e., with infinite crack tip stresses) was the Rayleigh wave speed of the medium. (An incorrect solution had been published by MOTT (1948), in which he obtained the erroneous result that the maximum speed of a tensile crack is some fixed fraction, typically about half the shear-wave speed of the material.) CRAGGS (1960) showed that the maximum rupture speed was the Rayleigh wave speed both for the tensile as well as the inplane shear crack. The maximum rupture speed for antiplane shear cracks was shown to be the shear-wave speed.

The presence of the singularity at the crack tip in perfectly brittle material leads to the result that for tensile and inplane shear cracks the maximum speed cannot exceed the Rayleigh wave speed; for antiplane shear cracks the maximum permissible rupture speed is the shear-wave speed. In numerical problems where the stress singularity at the crack edge is replaced by a large but finite stress, application of the critical stress level fracture criterion to the antiplane shear crack and the tension crack gave the same terminal rupture speeds as obtained for the analytical problems discussed above. But for inplane shear cracks, terminal speeds as high as the *P*-wave speed were found for relatively weak materials (ANDREWS, 1976b; DAS, 1976; DAS and AKI, 1977a).

The transition from sub-Rayleigh to super-shear speeds for inplane shear cracks is reproduced from DAS and AKI (1977a) in Figure 5. Similar results were found by ANDREWS (1976b) using Ida's criterion and a finite-difference method. The maximum permissible rupture speed has since been confirmed in numerous numerical studies. There has been much discussion recently on whether the transition from sub-Rayleigh to super-shear is sudden or smooth. With the computing power available in the late 1970s, this was impossible to resolve, but it could be resolved today, if so desired.

BURRIDGE *et al.* (1979) showed that even in the case of a perfectly brittle solid, cracks can propagate at $\sqrt{2}$ times the shear-wave speed. Very recent laboratory measurements (ROSAKIS *et al.*, 1999) confirm this.

Truly convincing observations of super-shear rupture speeds for earthquakes still remain elusive. In some reported cases it is not clear if the speed being measured is not the apparent rupture speed. Often such speeds are determined from one station close to the fault, or stations not close enough to the fault. Remembering the very unstable nature of the inverse problem of obtaining the fault rupture history from analysis of seismograms (KOSTROV and DAS, 1988; DAS and KOSTROV, 1990, 1994; DAS and SUHADOLC, 1996; DAS *et al.*, 1996; SARAO *et al.*, 1998), such velocities must not be unquestioningly accepted in situations without sufficient constraints (usually good station distribution and many three-component accelerograms). What is most important is that this theory has led to the search for such super-shear speeds, whereas previously such speeds would be considered impossible, and not considered at all.

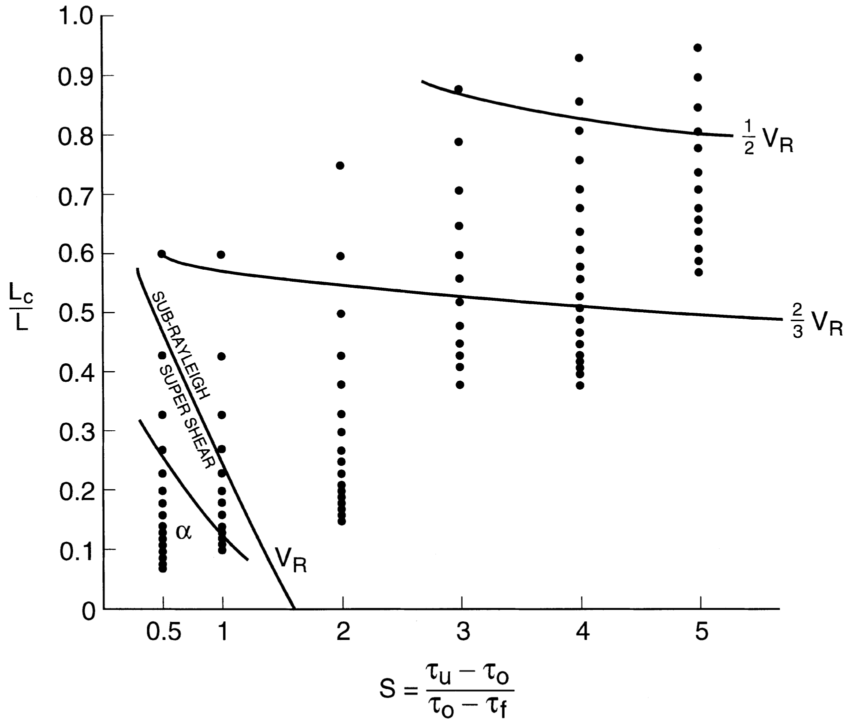

Figure 5
Transition from sub-Rayleigh to super-shear velocity for inplane cracks, (after DAS and AKI, 1977a), shown as contour plot of the crack tip velocity for different values of the fault strength parameter (S) against the dimensionless parameter L_c/L, where L_c is the initial crack length required for propagation to occur, i.e., the initial Griffith critical length, and L is the instantaneous crack length.

For the antiplane shear problem, the only wave speed in the problem is the shear-wave speed. But for the other two crack modes, P, S and Rayleigh waves all exist along the crack face. So why then does the tension crack not reach the P-wave speed even for the weakest materials? This can be explained by considering the Green functions for the two problems. The Green functions for the Mode II crack is such that the sign of the stress due to the body waves and the Rayleigh wave are the same, whereas for Mode I, the body wave stresses have opposite sign to the Rayleigh wave, and close the crack, inhibiting it from growing until the arrival of the Rayleigh wave. The tension crack, filled with fluid, was used by Aki and his co-workers, to model magma transport in volcanoes (AKI *et al.*, 1977).

Radiation from Spontaneous Faults

MADARIAGA (1976) considered the radiated body-wave pulses from a dynamic circular crack propagating at a constant velocity. He solved the problem numerically to determine the slip rate on the fault. Then, summing these fault slip rates in appropriate directions around the source (HASKELL, 1964), he obtained the pulse shape in those directions. He demonstrated that the far-field pulse shapes in different directions in a plane perpendicular to the crack can be used to estimate the fault dimension, if the rupture speed is known. In earlier kinematic models (SAVAGE, 1966; SATO and HIRASAWA, 1973) fault slip was stopped artificially, to find the expected pulse shapes at different stations. Madariaga's study from propagating faults in which the slip was stopped using a physically realistic criterion, first gave us insight into how fault slip stops on the different parts of a fault by "healing" information from the fault edges, and the resulting pulse shapes. MADARIAGA (1976) also considered the corner frequencies for faults propagating at different constant speeds and showed that the rupture speed affects the corner frequency. This implies, as DAS and AKI (1977b) demonstrated, that corner frequency is not a measure of fault size but of rupture time, and the two can be related if and only if the rupture speed [which in reality is variable as shown, for example, by KOSTROV (1966)] is known.

Complex Faulting Models

The barrier model. Using their numerical BIE method, DAS and AKI (1977b) considered the propagation of spontaneous 2-D faults on planes with variable values of *S*. The critical stress level fracture criterion was used in these calculations to determine how the crack advanced. This led to the development of what has now become known as the "barrier model" for heterogeneous faults, the barriers being regions of large *S*. In addition to the maximum permissible rupture speed, discussed above, other unexpected results were found, which cannot occur for the previously studied cracks with singularities at their edges. For example, a fault can jump over very strong regions and continue to propagate, leaving behind some unbroken regions. If these regions had very large *S* they still remained unbroken at the end of the rupture process, but if they had some intermediate values, the region was unbroken when the fracture front first jumped over it, but the concentration of stress on it during the dynamic rupture process led to its rupture.

What is most important in seismology is that the different kinds of rupture processes lead to different pulse shapes and hence these differences can potentially be inferred from seismograms. Figure 6 shows the "far-field" pulse shapes obtained by DAS and AKI (1977b) for smooth and rough faults. Their major conclusions are summarized below:

(i) The smooth fault and the fault with barriers that break during the dynamic process result in single earthquakes whereas the heterogeneous faults with unbroken barriers result in multiple shocks.

(ii) The time history of slip on the fault and the resulting far-field radiation is most complicated in the case when the initially unbreakable barrier eventually breaks. In

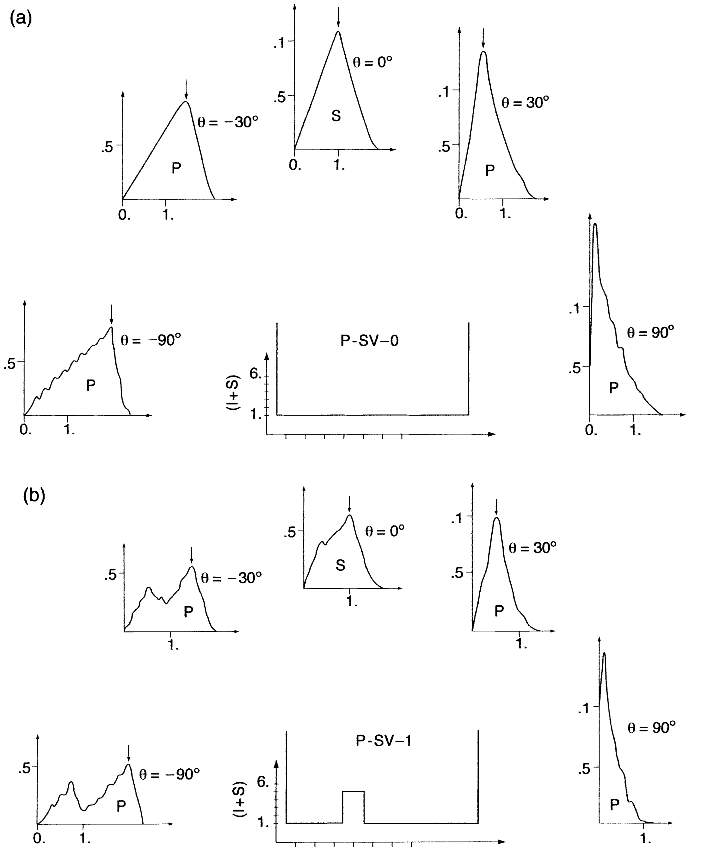

Figure 6

Pulse shapes for complex faulting models. (a), (b), (c) and (d) refer respectively to a fault without any complexity, a fault with one unbroken barrier, a fault with two unbroken barriers, and, a fault with two barriers which do not break when the rupture front passes it initially, but due to the increase of the stress on it caused by being surrounded by broken regions breaks while other parts of the fault are still fracturing dynamically.

this case the duration of the fracture and slipping process take longer than in the other cases for the same fault length.

(iii) The final slip on the fault and hence the seismic moment is largest for the smooth crack and smallest for the case of the fault with the most number of unbroken barriers. In the case of the barrier that eventually breaks, the final slip and moment are almost as large as that for the smooth fault. The slip for the fault with the largest number of unbreakable barriers has the most uniform value over the fault while the fault with no barriers at the end of the fracture process shows the largest amount of variation in slip distribution over the fault! This may explain why the uniform dislocation model (HASKELL, 1964) has often been able to explain observed

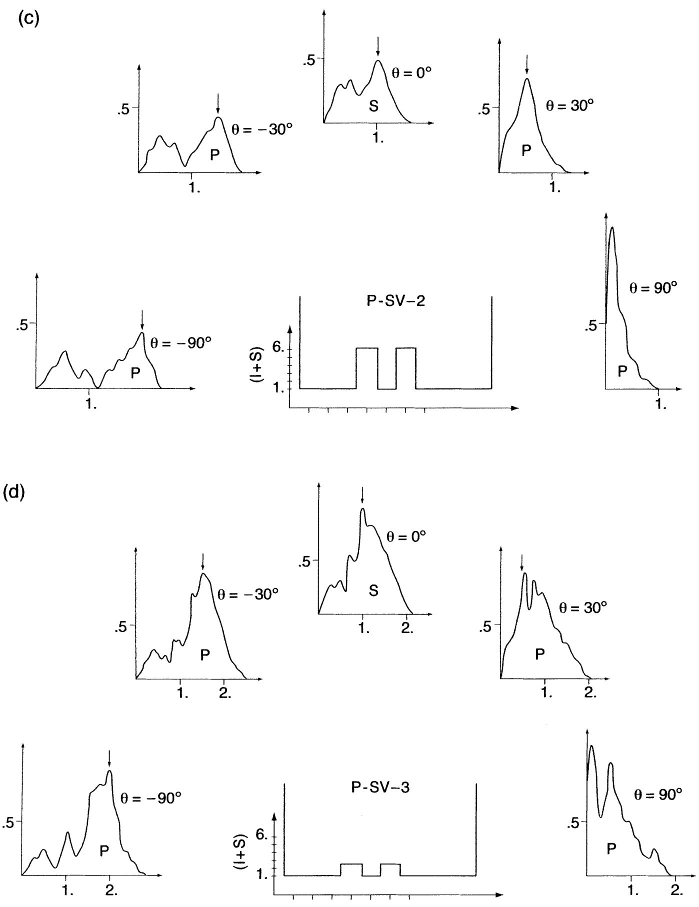

Figure 6c,d

overall features of seismograms satisfactorily, for example, BOUCHON's (1979) study of the 1966 Parkfield, California earthquake.

(iv) Clear directivity effects in the seismic radiation are seen in all cases, these effects being stronger for the fault with unbreakable barriers than for the smooth fault. However when the barriers eventually break the directivity effect is even weaker than that for the smooth fault.

(v) The time domain pulses are more sensitive to the complexity of the fracture process than the spectral shapes. In particular, when the barriers eventually break the pulses show complexity in all directions from the source but the amplitude spectra are not particularly revealing.

(vi) When the barriers remain unbroken, the spectra at the highest frequencies for which the numerical results are meaningful (this limit can be obtained by comparing the numerical solution for some simple case with an analytic solution, the spectra in all the cases plotted in this example being shown only up to the frequency where the numerical results are valid) have more energy than that for the smooth fault.

(vii) The corner frequency averaged over all directions from the source is unaffected by the presence of unbreakable barriers.

(viii) The stress drop averaged over the total fault length (including the barriers) is lower for the case with unbroken barriers than the other cases. In fact, there is a stress increase on these unbroken regions due to the earthquake. Thus, a complex earthquake with lower average stress drop can generate relatively higher frequency waves than a simple earthquake with relatively higher stress drop.

The idea that faults can jump over barriers was not immediately accepted, when first proposed by DAS and AKI (1977b), since in classical fracture mechanics with infinite crack tip stresses it cannot do so, as mentioned above. In the many observations since in which faults have been shown to jump across barriers, the barrier that is jumped over is relatively small, usually a few kilometers (AKI, 1979, 1980). A study of the great 1998 Antarctic plate earthquake shows that this earthquake jumped over a 70–100 km long barrier and kept propagating for another 60 km (HENRY *et al.*, 2000). This is similar to the case P-SV-1 illustrated in Figure 6b.

The asperity model. The basic idea of this model was first suggested by MADARIAGA (1979) and then by RUDNICKI and KANAMORI (1981). According to this model, an earthquake is caused by the failure of isolated, highly stressed regions of the fault, the rest of the fault having little or no resistance to slip (being partially broken and preslipped, say) and contributing little or no stress drop to the earthquake process. This results in a nonuniform stress drop over the fault. Since the regions without slip are able to withstand the high stresses concentrated on it until the moment of commencement of the earthquake, the model implies that the parameter σ_u for these regions is higher than that for the rest of the fault.

The observational support for complex faulting models came both from seismology and geology. Observations of multiple shocks on seismograms, the measured surface slip after large earthquakes, direct evidence from fractures on mine faces showing that faults are usually very complex with side-steps and highly deformed but unbroken ligaments in the step-over regions (SPOTTISWOODE and MCGARR, 1975; MCGARR *et al.*, 1979) all contributed to this. In spite of its idealizations, these models enhanced our understanding of the earthquake faulting process. It led to the characterization of barriers as being material (large *S*) or geometrical (when the fault plane deviated from planarity) by AKI (1979). It has led to the identification of barriers in the field by structural geologists and by seismologists in various locations world wide (LINDH and BOORE, 1981; KING and YIELDING, 1984; NABELEK and KING, 1985; SIBSON, 1986; BARKA and KADINSKY-

CADE, 1987; BRUHN *et al.*,1987; DAS, 1992, 1993; HENRY *et al.*, 2000, to name only a few among many such examples). Finally, the most convincing evidence that faults are heterogeneous not only near the surface of the Earth but also at the depths where the main faulting in an earthquake occurs is that aftershocks do occur at these depths. Effort is under way to to identify barriers along faults and to try to understand the origin and geochemical characteristics of barriers. The primary reason for this general interest is that earthquakes often nucleate and terminate at barriers.

How Faults Stop

A problem that was considered in the mid-1970s is how faults stop. ESHELBY (1969) had shown that the crack tip has no inertia. For seismological applications, this implies that fractures can start and stop suddenly. HUSSEINI *et al.* (1975) considered two possible ways in which faults can stop, either by encountering a large high strength region or by running into a "seismic gap," i.e., running out of available strain energy for fault propagation to continue. DAS (1976) demonstrated that these two methods of fracture arrest lead to different far-field spectra. This part of the work was never published by the author except as part of a thesis, but is reviewed by DMOWSKA and RICE (1985).

Summary of Developments since the Late 1970s

In 1980, Das continued the work started with Aki at MIT and developed the fully 3-D numerical BIE method. DAS (1981) applied this method to truly 3-D shear cracks, and confirmed the maximum rupture speeds in the purely mode II and III directions as being the same as found for the 2-D case, implying that in relatively strong materials, a circular crack remains circular as it grows, but on weak planes, they become elongated. The problem was further continued by Das, in collaboration with Kostrov, until the late 1980s.

Since the unbroken barrier with its high residual stress concentration can become the "asperity" of a future earthquake on the same fault, the radiation due to the fracturing of such an unbroken barrier was considered by DAS and KOSTROV (1983), who studied the dynamic fracture of a single circular asperity and showed that the rupture process is so complex, that the idea of rupture velocity becomes almost meaningless. Of the different cases studied by DAS and KOSTROV (1983), one is shown in Figure 7. Interestingly, for an elliptical asperity the rupture propagates as a very simple straight front (DAS and KOSTROV, 1985). The rupture of a pre-existing circular crack was also shown to be complex, and is shown in Figure 8, which is redrawn from KOSTROV and DAS (1988). DAS and KOSTROV (1986) also studied the rupture of a single asperity on a finite pre-broken fault and showed that its spectrum has the properties of a "slow" or "weak" earthquake. DAS (1986) compared the radiated field generated by the rupture of a circular crack and a circular asperity. We refer the reader to KOSTROV and DAS (1988) for a full treatment of these problems.

DAS and KOSTROV (1987) increased the efficiency of the 3-D BIE method, enabling the use of fine grids in the numerical problem. DAS and KOSTROV (1988) used this to study complex 3-D rupture on faults, and showed that all peaks on source time functions are not due to rupture of pre-existing asperities, and that "dynamic asperities" can appear and be seen on the source time function. Such dynamic concentrations of stresses, involving persistent "crack front waves" have been shown numerically to exist for 3-D tensile crack problems by MORRISSEY and RICE (1998), and have been shown analytically to exist by RAMANATHAN and FISCHER (1997) based on the 3-D perturbation solutions by MOVCHAN and WILLIS (1995). Earlier 3-D analytical solutions by RICE, BEN-ZION and KIM (1994) for acoustic wave (scalar model) problem, later also studied by WOOLFRIES and WILLIS (1999), showed a non-persistent, but only slowly decaying (algebraically in time) type of dynamic stress concentration. The 3-D perturbation solution has also been given

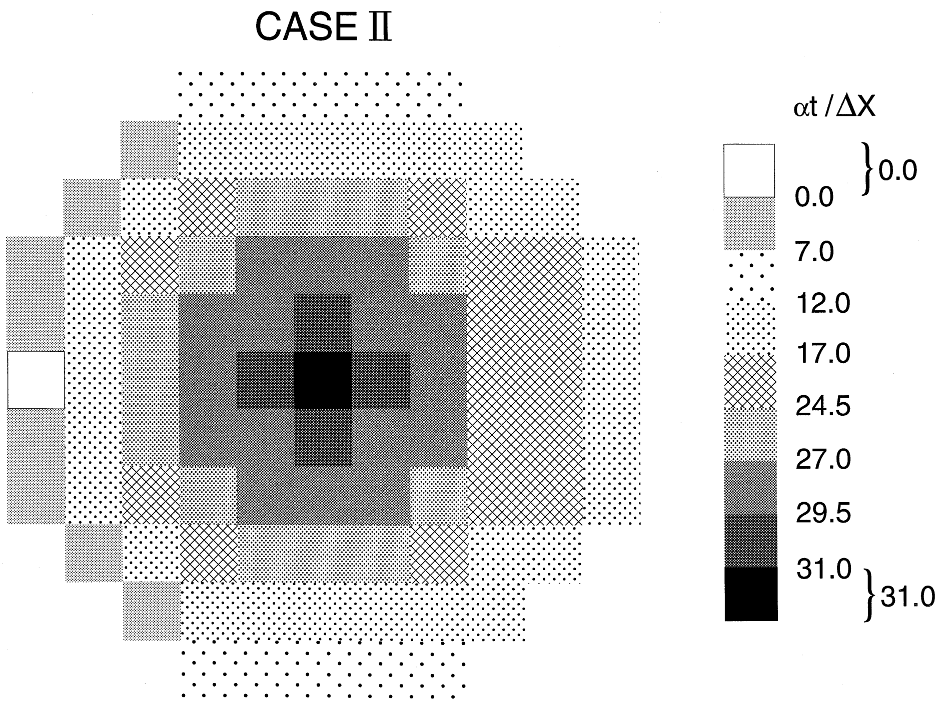

Figure 7
Rupture history for a pre-existing circular asperity on an infinite fault plane in shear, under a critical stress level fracture criterion.

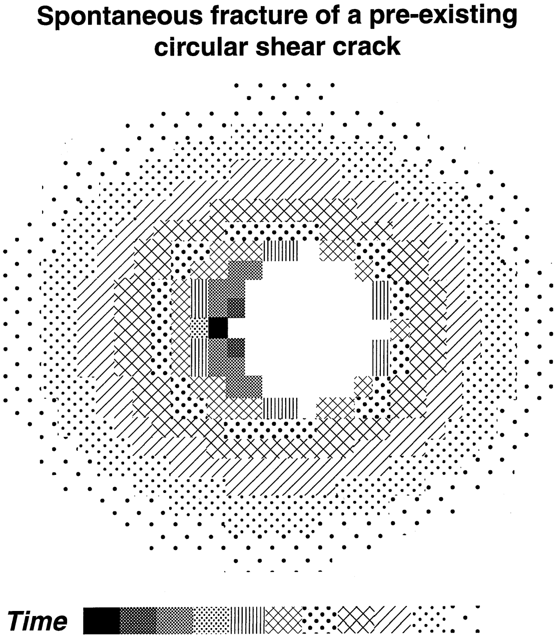

Figure 8
Rupture history for a pre-existing circular shear crack, under a critical stress level fracture criterion.

by Movchan and Willis (1995) for shear cracks, and by Willis and Movchan (1997) for cracks perturbed out of a plane, although it has not yet been established if and when these cases lead to persistent crack front waves.

Based on the work of Das and Kostrov (1988), Das (1987) created movies which demonstrated that the existence of heterogeneities leads to narrow zones of slip propagating across faults, which have since been called the "Heaton pulse." Efforts are underway to show that such narrow pulses arise from complex friction laws, but the work of Das and Kostrov (1988) demonstrate that simple Coulomb friction combined with fault heterogeneities can also result in such narrow slip pulses on the fault.

Further developments in the late 1980s and the 1990s continued with rate- and state-dependent friction laws incorporated into the models (Okubo, 1989). Some of this work is reviewed Madariaga and Peyrat (2000).

Finally, we can ask what the impact of the work done in the 1970s by Aki and coworkers has been? One can answer this using the following famous statement:

> "It is too soon to tell!"
>
> Chou-en Lai, former prime minister of China, on being asked what the effect of the French Revolution was on history.

Acknowledgements

I would like to thank the Department of Earth Sciences, University of Southern California for financial support to participate in this conference.

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(Received July 23, 2000, accepted February 27, 2002)