

## A Review of the Discrete Wavenumber Method

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*Abstract* — We present a review of the discrete wavenumber (DWN) method. The method, introduced by BOUCHON and AKI (1977), allows the simple and accurate calculation of the complete Green's functions for many problems in elastodynamics.

**Key words:** Wave propagation, synthetic seismograms, discrete wavenumber, earthquake ground motion.

### *Introduction*

The evaluation of Green's functions for acoustic or elastic media is an important problem in fields such as seismology or acoustics. Since the pioneering work of LAMB (1904), many approaches have been proposed to evaluate the response of elastic solids to excitation by transient point sources. The methods devised for the calculation of the Green's functions are, however, often very complex or, in many cases, only provide approximate solutions. The discrete wavenumber method, introduced by BOUCHON and AKI (1977), provides a way to accurately calculate the complete Green's functions for many problems with a minimum amount of mathematics.

The principle of the method may be traced back to Rayleigh, who demonstrated that waves reflected by a sinusoidally corrugated surface propagate only at discrete angles that he referred to as the orders of the spectrum (RAYLEIGH, 1896, 1907). The existence of discrete orders in the horizontal wavenumber spectrum is an immediate consequence of the periodicity of the reflecting surface. AKI and LARNER, in 1970, extended Rayleigh's approach to study the scattering of plane waves in the vicinity of a periodic irregular surface with the use of complex frequency. In the same way, the discrete wavenumber (DWN) method introduces a spatial periodicity of sources to discretize the radiated wave field, and relies on the Fourier transform in the complex frequency domain to calculate the Green's functions.

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### *Principle of the Method*

We shall begin with a short consideration of the 2-D case, as the principle of the method is easiest to describe in this case. The steady-state radiation from a line source in an infinite homogeneous medium can be represented as a cylindrical wave or, equivalently, as a continuous superposition of homogeneous and inhomogeneous plane waves. Therefore, denoting by  $x$  and  $z$  the horizontal and vertical axes in the plane normal to the line source, any observable such as displacement or stress can be written in the form

$$F(x, z; \omega) = e^{i\omega t} \int_{-\infty}^{\infty} f(k, z) e^{-ikx} dk \quad (1)$$

where  $\omega$  is the frequency and  $k$  is called the horizontal wavenumber. Equation (1) still holds for an extended two-dimensional source located in a medium which is homogeneous in any horizontal plane.

When the medium is finite or vertically heterogeneous, the integral kernel has poles and singularities, and the integration over the horizontal wavenumber becomes mathematically and numerically complicated. One simple way around these difficulties is to replace the single-source problem, whose solution is expressed by (1), by a multiple-source problem where sources are periodically distributed along the  $x$  axis. Then, equation (1) is replaced by:

$$G(x, z; \omega) = \int_{-\infty}^{\infty} f(k, z) e^{-ikx} \sum_{m=-\infty}^{\infty} e^{ikmL} dk \quad (2)$$

where  $L$  is the periodicity source interval and the  $e^{i\omega t}$  time dependence is understood. Equation (2) reduces to:

$$G(x, z; \omega) = \frac{2\pi}{L} \sum_{n=-\infty}^{\infty} f(k_n, z) e^{-ik_n x} \quad (3)$$

with

$$k_n = \frac{2\pi}{L} n$$

which in turn, if the series converges, can be approximated by the finite sum equation

$$G(x, z; \omega) = \frac{2\pi}{L} \sum_{n=-N}^N f(k_n, z) e^{-ik_n x} . \quad (4)$$

In moving from equation (1) to equation (4), we have greatly reduced the calculation. In so doing, however, we have changed the problem from one of a single

source, to one involving an infinite number of periodic sources, as illustrated in Figure 1. The DWN method calculates equation (4), that is  $G(x, z; \omega)$ , instead of evaluating equation (1).

The second stage of the method is to retrieve the single-source solution from the multiple-source problem that we have solved in the frequency domain. This would be straightforward if we could calculate the continuous Fourier transform of  $G$ , as we could then isolate the single source solution in the time domain, provided that we have chosen an appropriate value for  $L$ . In practice, however, we can only calculate  $G$  for a certain number of frequencies and use the discrete Fourier transform to obtain the time domain solution. Thus, on one hand we deal with a signal which has an infinite time response (because of the infinite set of sources), while on the other hand, we use the discrete Fourier transform, which yields a signal of finite duration  $T = 2\pi/\Delta\omega$ , where  $\Delta\omega$  is the angular frequency sampling used in calculating  $G$ . This can indeed be accomplished by performing the Fourier transform in the complex frequency domain:

$$g(x, z, t) = \int_{-\infty+i\omega_I}^{\infty+i\omega_I} G(x, z; \omega) e^{i\omega t} d\omega \tag{5}$$

where  $\omega_I$  denotes the constant imaginary part of the frequency and is chosen such that

$$e^{\omega_I T} \ll 1 \tag{6}$$

This last equation insures the attenuation, over the time window  $T$ , of the previously infinite time response solution. Thus, provided that we have chosen  $L$  large enough so that no disturbance arrives at the receiver  $(x, z)$  from the next closest source in the time window of interest  $T$ , the time-domain single-source solution  $f(x, z; t)$  is obtained from the frequency-domain multiple-source calculation  $G(x, z; \omega)$  by

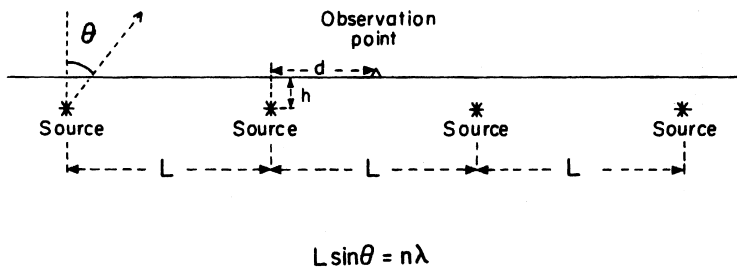


Figure 1  
Physical interpretation of the DWN method. The single source is replaced by an infinite array of sources distributed horizontally at equal interval  $L$ . For a given radiation wavelength  $\lambda$  corresponding to a specific frequency of excitation, the elastic energy is radiated in discrete directions  $\theta$  only.

$$f(x, z; t) = e^{-\omega_I t} \int_{-\infty}^{\infty} G(x, z; \omega) e^{i\omega_R t} d\omega_R \quad (7)$$

where the integral is computed by using the FFT.

Equation (6) shows that  $\omega_I$  is only a function of the length of the time window  $T$  considered. The results should not be sensitive to the particular value of  $\omega_I$  chosen, as long as it provides enough attenuation for the disturbances which arrive after the time window of interest  $T$  to be negligible. Values in the range:

$$\omega_I = \left[ -\frac{\pi}{T}, -\frac{2\pi}{T} \right] \quad (8)$$

are recommended for most applications.

It is worth noting here that disturbances which arrive in the time range  $[T, 2T]$  will be attenuated by  $e^{\omega_I T}$ , while disturbances in the time range  $[2T, 3T]$  will be attenuated by  $e^{2\omega_I T}$ , and so on. The choice of  $\omega_I$  may also be justified by the fact that the frequency spectrum  $G(\omega)$  is not discrete, as would be the case with real frequencies, but is continuous with a bandwidth proportional to  $\omega_I$  (LARNER, 1970). Choosing values in the range of relation (8) implies that the bandwidth of the spectral lines is of the order of the frequency interval. Thus, the calculated signals may also be considered as resulting from a nearly continuous sampling of the frequency domain.

In Figure 2, we present a comparison of the numerical results obtained through these equations with an analytical solution. The case considered involves an explosive line source in a half-space, as it is one of the rare cases where an analytical solution exists (GARVIN, 1956). The comparison shows the great accuracy of the DWN method.

### *Discretization in Various Coordinate Systems*

The simplest type of elastic source in three-dimensions is an isotropic point source. The wave field radiated by such a source can be conveniently represented by the displacement potential, which, for a steady-state excitation, is given by:

$$\phi(R; \omega) = \frac{-V_S(\omega)}{4\pi R} e^{i\omega(t-R/\alpha)} \quad (9)$$

where  $V_S$  is the volume change at the source and  $\alpha$  denotes the compressional wave velocity.

In the shallow earth, where boundaries are nearly horizontal and where the medium properties change primarily with depth, using this spherical wave representation would be most cumbersome, so we must express the wave field in more appropriate coordinate systems. One possibility is to use a Cartesian system with the

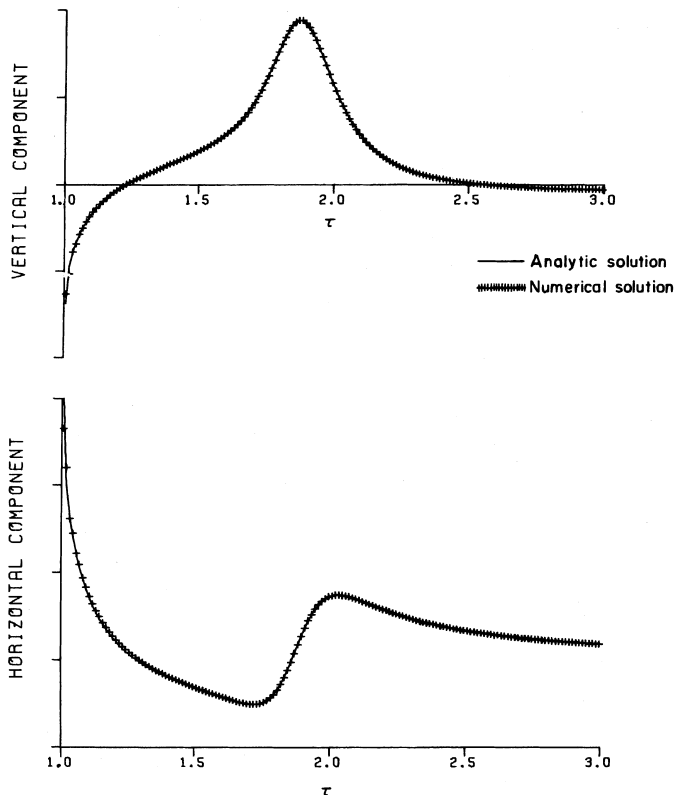


Figure 2

Comparison between numerical and analytical solutions for the surface displacement due to a buried explosive line source with step-function time dependence. Computations are made for a Poisson ratio of 0.25 and a ratio of distance  $R$  to source depth equal to 10.  $\tau = t\alpha/R$ , where  $t$  is time and  $\alpha$  is the compressional wave velocity. The analytical displacements are infinite at the time of  $P$ -wave arrival ( $\tau = 1$ ).

$z$  axis running vertically. In such a system, the wave field is expressed as a double integral over the two components of the horizontal wavenumber,  $k_x$  and  $k_y$ , through the Weyl integral (LAMB, 1904; AKI and RICHARDS, 1980):

$$\phi(x, y, z; \omega) = \frac{iV_S(\omega)}{8\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{v} e^{-iv|z|} e^{-ik_x x} e^{-ik_y y} dk_x dk_y \tag{10}$$

with

$$v = \sqrt{\frac{\omega^2}{\alpha^2} - k_x^2 - k_y^2}, \quad \text{Im}(v) < 0 ,$$

where the origin of the coordinate system is taken at the source, and the  $e^{i\omega t}$  dependence is understood.

The generalization of the previous results from 2-D to 3-D is straightforward and leads to the following expressions (BOUCHON, 1979):

$$\phi(x, y, z; \omega) = \frac{iV_S(\omega)}{2L_x L_y} \sum_{n_x=-N_x}^{N_x} \sum_{n_y=-N_y}^{N_y} \frac{1}{v} e^{-iv|z|} e^{-ik_{n_x}x} e^{-ik_{n_y}y} \quad (11)$$

with

$$k_{n_x} = \frac{2\pi}{L_x} n_x, \quad k_{n_y} = \frac{2\pi}{L_y} n_y$$

for which the corresponding multiple-source problem is a periodic array of sources distributed at equal intervals  $L_x$  in the  $x$  direction, and  $L_y$  in the  $y$  direction.

In many wave propagation problems, the elastic wave field may also be conveniently expressed in a cylindrical coordinate system with  $z$  as the vertical axis. The wave field is then represented as an integral over the horizontal wavenumber through the Sommerfeld integral:

$$\phi(r, z; \omega) = \frac{iV_S(\omega)}{4\pi} \int_0^{\infty} \frac{k}{v} J_0(kr) e^{-iv|z|} dk \quad (12)$$

with

$$v = \sqrt{\frac{\omega^2}{\alpha^2} - k^2}, \quad \text{Im}(v) < 0$$

and where  $J_0$  denotes the zeroth order Bessel function.

The discretization of this equation can also be achieved by replacing the single-source by a periodic arrangement of sources which, in this case, consists of the original point source plus an infinite array of circular sources centered around the point source and distributed at equal radial interval  $L$  (BOUCHON, 1981). This physical arrangement leads to:

$$\phi(r, z; \omega) = \frac{iV_S(\omega)}{2} \sum_{n=0}^N \frac{k_n}{v_n} J_0(k_n r) e^{-iv_n|z|} \quad (13)$$

with

$$k_n = \frac{2\pi}{L} n .$$

The comparison between the two geometric source arrangements resulting in discretizations (11) and (13) is shown in Figure 3.

Once the source radiation has been decomposed, through equations (4), (11), or (13), into a superposition of waves propagating with discrete wavenumbers, the effect

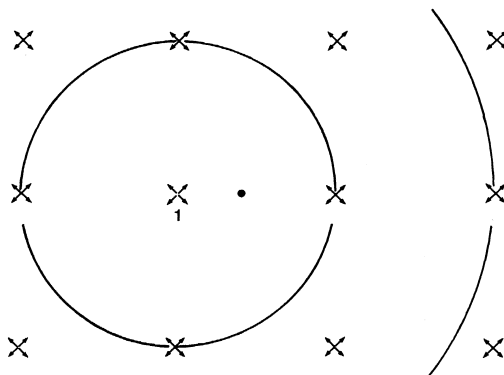


Figure 3

Geometries of source-receiver configurations leading to the discretization: a circular source array for the  $k$  discretization scheme and a rectangular network for the  $(k_x, k_y)$  discretization method. Source 1 is the original single-source problem. The black dot shows the receiver location.

of plane boundaries and flat layers is taken into account by using, for each horizontal wavenumber component, the corresponding plane-wave reflection and transmission coefficients at the medium surface and interfaces, and summing up all the wavenumber contributions. This is best done by calculating, for each wavenumber involved in the source radiation, the corresponding reflectivity and transmissivity matrices of the layered medium (KENNETT, 1974; KENNETT and KERRY, 1979; MÜLLER, 1985). The truncation of the wavenumber series is easily determined for each frequency by a simple convergence criterion which compares the new wavenumber contribution to the current sum of the series, and stops the calculation when the new contribution becomes negligible.

The accuracy of the two discretization schemes (11) and (13) can be measured by comparing synthetic seismograms obtained using these equations, as the two schemes are independent. This is done in Figure 4, where the similitude of the results demonstrates the accuracy of the DWN method. In most applications, the  $k$  discretization scheme will be preferred over the  $k_x, k_y$  scheme because it involves only one summation and the resulting calculation is faster. One such application is displayed in Figure 5.

For other types of problems, other schemes of discretization may be devised. For instance, in the case of a source in a borehole, common in exploration geophysics, it is convenient to use, for equation (9), the expression:

$$\phi(r, z; \omega) = \frac{-V_S(\omega)}{4\pi^2} \int_{-\infty}^{\infty} K_0(vr) e^{-ikz} dk \quad (14)$$

with

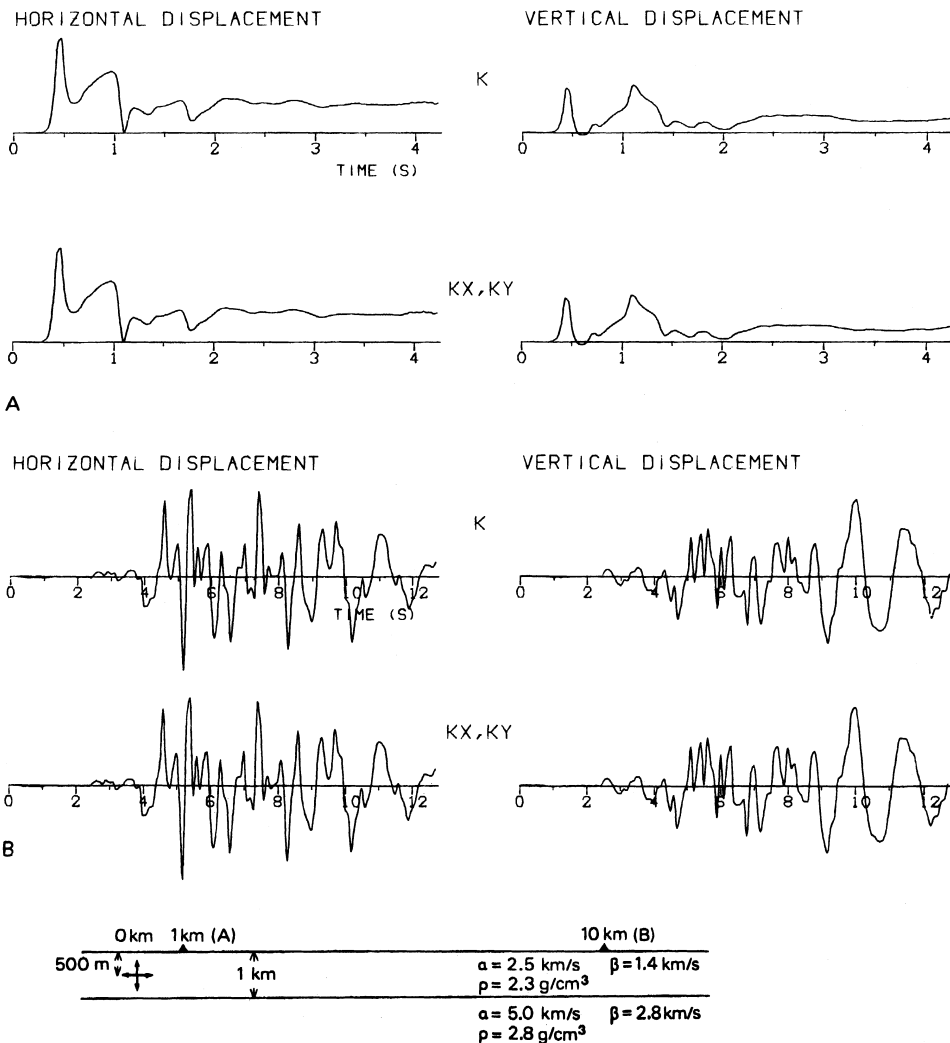


Figure 4

Comparison of surface displacements obtained using the  $k$  and  $(k_x, k_y)$  discretization schemes for an explosion in a layer over a half-space model. The source-time function is  $\frac{1}{2}[1 + \tan h(t/t_0)]$  with  $t_0 = 0.1$  s. First motions are up and away from the source.

$$v = \sqrt{\frac{\omega^2}{\alpha^2} - k^2}, \quad \text{Im}(v) < 0 ,$$

where  $(r, z)$  are cylindrical coordinates centered at the source and  $z$  runs along the borehole axis,  $k$  is now the vertical wavenumber (in the case of a vertical borehole), and where  $K_0$  denotes the zeroth-order modified Bessel function of the second kind.



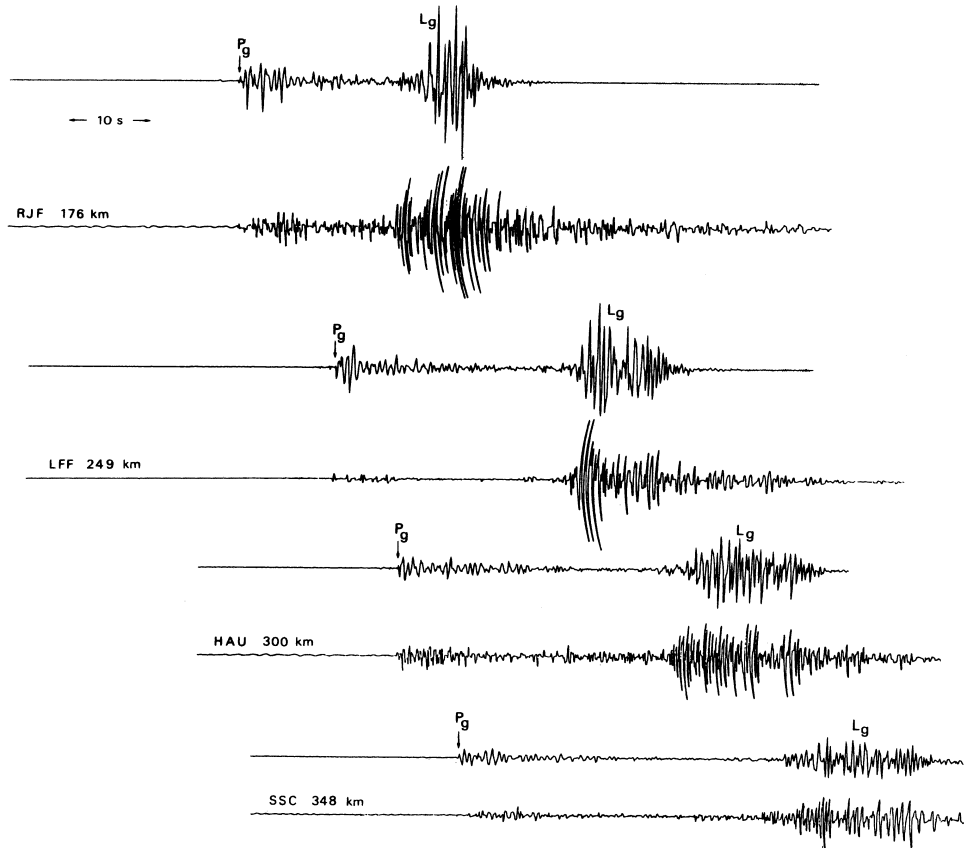


Figure 5

Comparison of the vertical short-period seismograms synthesized (upper trace) and observed (lower trace) at four stations for a small earthquake in central France. The epicentral distance of each station is indicated. The propagation model used in the calculation consists of four crustal layers overlaying a mantle half-space. The source is a double-couple point with the mechanism of the earthquake and located at a depth of 10 km. The slip time dependence is a ramp function with a rise time of 0.2 s (after BOUCHON, 1982a).

The discretization of this expression, which was introduced by CHENG and TOKSÖZ (1981), yields:

$$\phi(r, z; \omega) = \frac{-V_S(\omega)}{2\pi L} \sum_{n=-N}^N K_0(v_n r) e^{-ik_n z} \tag{15}$$

with

$$k_n = \frac{2\pi}{L} n$$

and corresponds to a periodic arrangement of point sources distributed at interval  $L$  along the  $z$  axis.

Expression (15) is convenient to use in a borehole environment because, in this form, cylindrical boundaries of the borehole, tubing, mud casing, and/or borehole tool can be taken into account through propagator matrices or reflectivity/transmissivity matrices similar to the ones in flat layer media. An example of such a calculation is displayed in Figure 6.

### *Case of a Generalized and Extended Source*

We now consider the case where the point source is a force with Cartesian components  $(F_x, F_y, F_z)$ , and we express its radiation in a discretized form similar to (13). We assume again that the cylindrical coordinate system is centered at the source

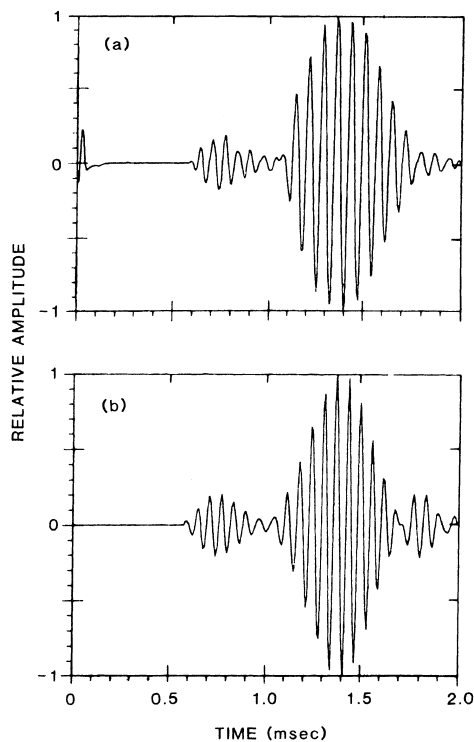


Figure 6

A comparison between (a) actual and (b) synthetic full waveform acoustic log microseismograms in a limestone formation. The source is a pressure point located in a fluid-filled cylindrical borehole. Parameters used are  $\alpha = 5.95$  km/s,  $\beta = 3.05$  km/s,  $\rho = 2.3$  for the geological formation, and  $\alpha = 1.83$  km/s,  $\rho = 1.2$  for the fluid. The borehole radius is 6.7 cm. The synthetic microseismogram is calculated by discretizing the source radiation in the vertical wavenumber domain (after CHENG *et al.*, 1982).

and that the  $z$  axis is vertical. We have for the compressional and rotational potentials:

$$\begin{aligned}
 \phi(r, \theta, z; \omega) &= \frac{1}{2L\rho\omega^2} \left[ \operatorname{sgn}(z)F_z \sum_{n=0}^N k_n J_0(k_n r) e^{-i\nu_n |z|} \right. \\
 &\quad \left. - i(F_x \cos \theta + F_y \sin \theta) \sum_{n=0}^N \frac{k_n^2}{\nu_n} J_1(k_n r) e^{-i\nu_n |z|} \right] \\
 \psi(r, \theta, z; \omega) &= \frac{1}{2L\rho\omega^2} \left[ -iF_z \sum_{n=0}^N \frac{k_n}{\gamma_n} J_0(k_n r) e^{-i\gamma_n |z|} \right. \\
 &\quad \left. + \operatorname{sgn}(z)(F_x \cos \theta + F_y \sin \theta) \sum_{n=0}^N J_1(k_n r) e^{-i\gamma_n |z|} \right] \tag{16} \\
 \chi(r, \theta, z; \omega) &= i \frac{F_y \cos \theta - F_x \sin \theta}{2L\rho\beta^2} \sum_{n=0}^N \frac{1}{\gamma_n} J_1(k_n r) e^{-i\gamma_n |z|}
 \end{aligned}$$

with

$$\gamma_n = \sqrt{\frac{\omega^2}{\beta^2} - k_n^2}, \quad \operatorname{Im}(\gamma_n) < 0$$

and

$$\operatorname{sgn}(z) = 1 \text{ for } z > 0, \quad \operatorname{sgn}(z) = -1 \text{ for } z < 0.$$

where  $\rho$  is the density,  $\beta$  the shear-wave velocity, and  $J_1$  is the Bessel function of the first order.

Any type of elastic source can be represented by a combination of point forces. In particular, a generalized point source is commonly represented in seismology by its moment tensor  $m_{ij}$  where  $m_{xx}$ ,  $m_{yy}$ , and  $m_{zz}$  represent three force dipoles oriented along the Cartesian axes, while  $m_{xy} = m_{yx}$ ,  $m_{xz} = m_{zx}$ , and  $m_{yz} = m_{zy}$  are double couples with force oriented along the first axis index and arm along the second axis index. Expressions for the radiation from an arbitrary moment tensor source can then be obtained by linear operations on equations (16).

Of particular interest is the radiation from a double-couple source, as such a body source is equivalent to a point of shear dislocation. Denoting by  $(s_x, s_y, s_z)$  the components of the unit vector in the slip direction and by  $(n_x, n_y, n_z)$  those of the normal to the fault, the corresponding moment tensor components are:

$$m_{ij} = -\mu \operatorname{slip}(\omega) \Delta S (s_i n_j + s_j n_i) \tag{17}$$

where  $\mu$  is the rigidity and  $\Delta S$  is the elementary fault surface on which slip occurs.

The simplest way to calculate the elastic radiation from an extended source is usually to represent the source by a superposition of elementary point sources.

Although analytical expressions of the radiation can sometimes be derived in the frequency-wavenumber domain for particular cases, the point-source superposition is generally more versatile. In the case of an earthquake, for instance, the fault can be discretized into a two-dimensional array of double-couple points distributed on the fault plane at a spacing smaller than the shortest wavelength considered in the problem. Each point radiates with a phase delay  $e^{-i\omega t_r}$ , where  $t_r$  denotes the time for rupture to propagate from the hypocenter to the particular fault location. Slip amplitude and duration may vary at each point. The summation of all the elementary contributions is done in the frequency-wavenumber domain, and does not affect the calculation of the reflection/transmission and reflectivity/transmissivity matrices. As an example, the simulation of the ground motion produced during the 1999 Izmit earthquake is presented in Figure 7. For this calculation, the 135 km long fault is represented by 10,800 double-couple points uniformly distributed at a spacing of 500 m in the horizontal and vertical directions. One important aspect of the DWN method, which is illustrated in this figure, is that the method calculates the complete elastic wave field, including both static and dynamic contributions.

#### *Applications and Extensions of the Method*

The DWN method has been successfully tested against analytical solutions and other techniques (e.g., YAO and HARKRIDER, 1983; BEN-ZION and AKI, 1990) and has been extensively used to check the accuracy of other methods like finite-differences, finite-elements, ray methods, mode summation, or pseudo-spectral techniques (e.g., STEPHEN *et al.*, 1985; SAIKIA and HERRMANN, 1986; BEYDOUN and KEHO, 1987; MAUPIN, 1996; AOI and FUJIWARA, 1999; MOCZO *et al.*, 1999).

It has been used to study a variety of problems in elastodynamics where the calculation of Green's functions is required. Many of the applications have been carried out using the numerical code of COUTANT (1990).

Applications include problems in seismic exploration (CHENG and TOKSÖZ, 1981; CHENG *et al.*, 1982; DIETRICH and BOUCHON, 1985a,b; SCHMITT and BOUCHON, 1985; DIETRICH, 1988; SCHMITT, 1988b; CHENG, 1989; JEAN and BOUCHON, 1991; MEREDITH *et al.*, 1993; GIBSON, 1994; FALK *et al.*, 1996; HAARTSEN and PRIDE, 1997), earthquake seismology (CAMPILLO *et al.*, 1984, 1985; SAIKIA and HERRMANN, 1987; GARIEL *et al.*, 1990, 1991; OU and HERRMANN, 1990; CHIN and AKI, 1991; FUKUSHIMA *et al.*, 1995; PLICKA and ZAHRADNIK, 1998; PLICKA *et al.*, 1998), microseismicity studies (BERNARD and ZOLLO, 1989; GOT and FRÉCHET, 1993; JONGMANS and MALIN, 1995; ZOLLO *et al.*, 1995; ZOLLO and IANNACCONE, 1996; THEODULIDIS *et al.*, 1996), broadband modeling of local and regional seismograms (HERRMANN *et al.*, 1980; EBERHART-PHILLIPS *et al.*, 1981; BOUCHON, 1982a; CHRISTOFFERSSON *et al.*, 1988; BERTIL *et al.*, 1989; PAUL and NICOLLIN, 1989; ROBERTS and

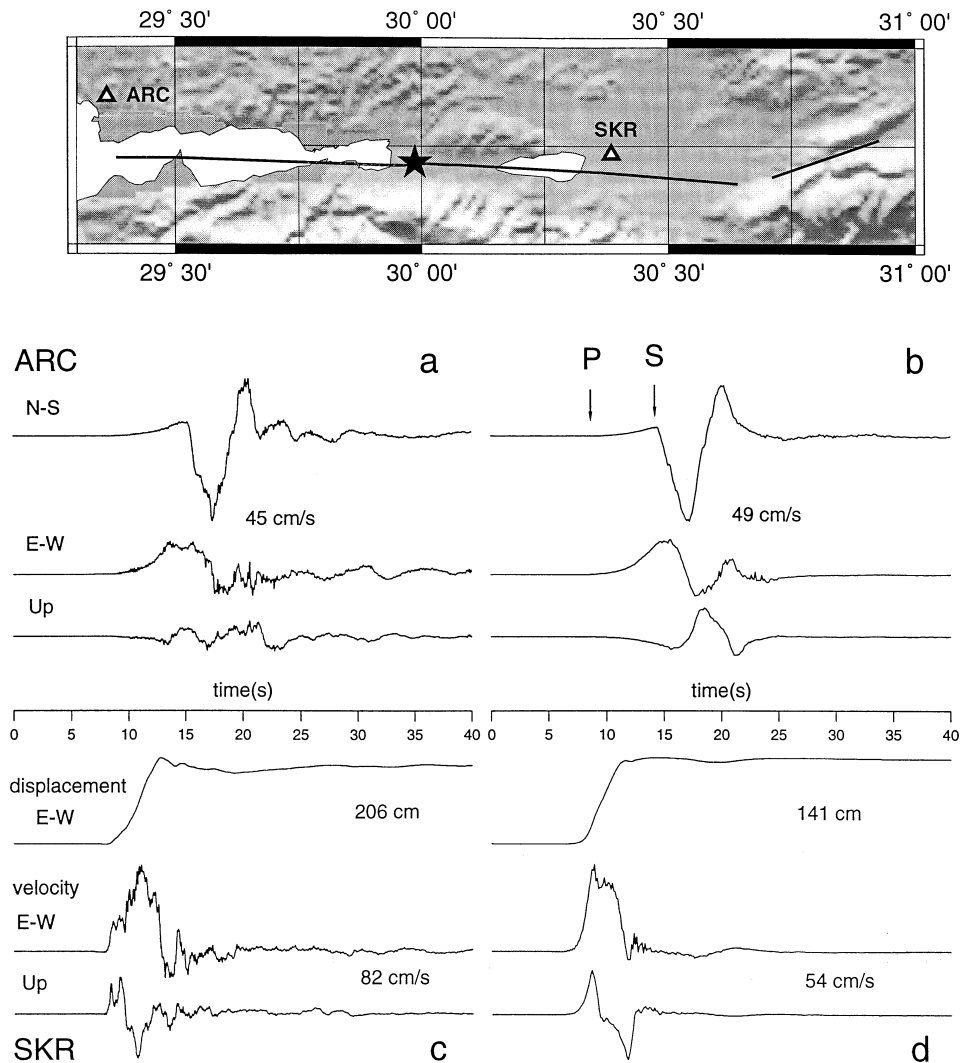


Figure 7

Comparison between recorded and calculated ground motion during the 1999 Izmit, Turkey, earthquake. (Top) Map of the surface rupture of the earthquake (solid line). The symbols indicate the location of the epicenter (star) and of the recording stations (triangles). (Middle) Ground velocity recorded (a) and calculated (b) at ARC. (Bottom) Displacement and velocity recorded (c) and calculated (d) at SKR. The numerical values indicated give the peak amplitudes of the observed/calculated velocity/displacement. All the traces start at the origin time of the rupture. The N-S component was inoperative at SKR. The fault is a vertically dipping strike-slip fault which follows the surface breaks shown on the map and extends from the surface down to 20 km. Rupture starts at the hypocenter, located at a depth of 17 km, and propagates toward the west at 3 km/s and toward the east at 4.7 km/s. Slip varies along the fault strike according to surface observations. Slip duration everywhere is 3 s. The lower than observed, peak values at SKR indicate a larger fault slip at depth near this station than the one observed at the surface.

CHRISTOFFERSSON, 1990; FUKUYAMA *et al.*, 1991; PEDERSEN and CAMPILLO, 1991; TOKSÖZ *et al.*, 1990; CAMPILLO and PAUL, 1992; TAKEO, 1992; CAMPILLO and ARCHULETA, 1993; STEIDL *et al.*, 1996; SAIKIA and HELMBERGER, 1997; SINGH *et al.*, 1997, 1999a,b; TSELENTIS and ZAHRADNIK, 2000), moment tensor inversion (CRUSEM and CARISTAN, 1992; NISHIMURA *et al.*, 2000; SINGH *et al.*, 2000; TEYSSONEYRE *et al.*, 2001), scattering (ZENG *et al.*, 1991; ZENG, 1993; MOINET and DIETRICH, 1998), fault zone effects (BEN-ZION, 1998), ground motion near earthquakes (AKI *et al.*, 1978; BOUCHON, 1980a,b, 1982b; CAMPILLO, 1983; BERNARD and MADARIAGA, 1984; MENDEZ and LUCO, 1988; CAMPILLO *et al.*, 1989; GARIEL and CAMPILLO, 1989; BARD *et al.*, 1992; TAKEO and ITO, 1997; TAKEO and KANAMORI, 1997; PEYRAT *et al.*, 2001), earthquake fault tomography (TAKEO, 1987, 1988; FUKUYAMA and MIKUMO, 1993; TAKEO *et al.*, 1993; COTTON and CAMPILLO, 1994, 1995a,b; SEKIGUCHI *et al.*, 1996, 2000; COTTON *et al.*, 1996; IDE *et al.*, 1996; MENDOZA and FUKUYAMA, 1996; IDE and TAKEO, 1996, 1997; COURBOULEX *et al.*, 1997; NAKAYAMA and TAKEO, 1997; OGLESBY and ARCHULETA, 1997; HERNANDEZ *et al.*, 1999; REBOLLAR *et al.*, 1999; QUINTANAR *et al.*, 1999), stress calculations (COTTON and COUTANT, 1997; BELARDINELLI *et al.*, 1999) and volcanology (CHOUET, 1981, 1982, 1985; AKI, 1984; CHOUET and JULIAN, 1985; TAKEO, 1990; NISHIMURA and HAMAGUCHI, 1993; GOLDSTEIN and CHOUET, 1994; UHIRA *et al.*, 1994; NISHIMURA, 1995; NISHIMURA *et al.*, 1995).

The DWN method has been extended to include anisotropic media (MANDAL and MITCHELL, 1986; MANDAL and TOKSÖZ, 1990, 1991; MANDAL, 1991) and two-phase media (BOUTIN *et al.*, 1987; SCHMITT, 1988a, 1990; SCHMITT *et al.*, 1988a, 1988b).

The discrete wavenumber formalism has also been extended to model wave propagation in 2-D or 3-D media through formulations based on boundary integral equations (BOUCHON, 1985; CAMPILLO and BOUCHON, 1985; CAMPILLO, 1987; PAUL and CAMPILLO, 1988; COUTANT, 1989; GAFFET and BOUCHON, 1989, 1991; BOUCHON *et al.*, 1989, 1996; AXILROD and FERGUSON, 1990; CAMPILLO *et al.*, 1993; CHAZALON *et al.*, 1993; GAFFET *et al.*, 1994; GIBSON and CAMPILLO, 1994; HAARTSEN *et al.*, 1994; GAFFET, 1995; KARABULUT and FERGUSON, 1996; TAKENAKA *et al.*, 1996; SHAPIRO *et al.*, 1996), boundary elements (KAWASE, 1988; KAWASE and AKI, 1989, 1990; KIM and PAPAGEORGIOU, 1993; BOUCHON and COUTANT, 1994; DONG *et al.*, 1995; PAPAGEORGIOU and PEI, 1998; ZHANG *et al.*, 1998; FU and WU, 2001), or generalized reflection/transmission matrices (CHEN, 1990, 1995, 1996). Hybrid methods of calculation, combining the method with finite-difference or finite-element methods, have also been developed to study the propagation of seismic waves in complex geological structures (ZAHRADNIK, 1995; ZAHRADNIK and MOCZO, 1996; MOCZO *et al.*, 1997; RIEPL *et al.*, 2000).

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