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Calculation of Synthetic Seismograms with Gaussian Beams

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Abstract — In this paper, an overview of the calculation of synthetic seismograms using the Gaussian beam method is presented accompanied by some representative applications and new extensions of the method. Since caustics are a frequent occurrence in seismic wave propagation, modifications to ray theory are often necessary. In the Gaussian beam method, a summation of paraxial Gaussian beams is used to describe the propagation of high-frequency wave fields in smoothly varying inhomogeneous media. Since the beam components are always nonsingular, the method provides stable results over a range of beam parameters. The method has been shown, however, to perform better for some problems when different combinations of beam parameters are used. Nonetheless, with a better understanding of the method as well as new extensions, the summation of Gaussian beams will continue to be a useful tool for the modeling of high-frequency seismic waves in heterogeneous media.

Key words: Synthetic seismograms, Gaussian beams.

Introduction

The Gaussian beam method is an asymptotic method for the computation of wave fields in smoothly varying inhomogeneous media, and was proposed by POPOV (1981, 1982) based on an earlier work of BABICH and PANKRATOVA (1973). The method was first applied by POPOV *et al.* (1980), KATCHALOV and POPOV (1981) and ČERVENÝ *et al.* (1982) to describe high-frequency seismic wave fields by the summation of paraxial Gaussian beams. One of the advantages of the method is that the individual Gaussian beams have no singularities either at caustics in the spatial domain or at pseudo-caustics in the wavenumber domain. Although caustics and pseudo-caustics are generally at different locations, methods such as the Maslov method require them to be well separated. The lack of singularities of the individual beams assures that the summation of Gaussian beams is regular everywhere. Another advantage of the Gaussian beam method is that it naturally introduces smoothing effects and is therefore not as sensitive to model parameterizations as the ray method. Finally, the Gaussian beam method does not require two-point ray tracing, unlike the ray method.

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An overview of the Gaussian beam method is first given, including a description of paraxial Gaussian beams, the superposition of Gaussian beams for the construction of more general wave fields, and the selection of beam parameters. Next, a representative list of applications of the method is presented. Finally, several extensions of the original method that have been proposed are described.

Rays and Linearized Rays

A high-frequency wave solution connected to a given ray can be written in ray centered coordinates (q_1, q_2, s) as

$$\vec{u}(q_I, s) = \vec{U}(s)e^{-i\omega(t-\tau(q_I,s))} \ , \tag{1}$$

where $I = 1, 2$ and the phase time up to second order is

$$\tau(q_I, s) = \tau(s) + \frac{1}{2}\vec{q}^T M(s)\vec{q} \ , \tag{2}$$

where $M(s)$ is a 2×2 matrix related to the local curvature of the wavefront by $K(s) = vM(s)$, and v is the medium velocity along the ray. $M(s)$ is obtained by solving a matrix Riccati equation along the ray, and can be decomposed into two submatrices as $M(s) = P(s)Q^{-1}(s)$ (Popov and Pšenčík, 1978; Červený and Hron, 1980).

The matrices $P(s)$ and $Q(s)$ are 2×2 matrices which are solutions of the first-order dynamic or linearized ray equations which can be written as

$$\frac{dX}{ds} = AX, \quad \text{where} \quad A = \begin{pmatrix} 0 & v\delta_{IJ} \\ -v^{-2}v_{,IJ} & 0 \end{pmatrix}, \quad X(s) = \begin{pmatrix} Q(s) \\ P(s) \end{pmatrix} \ , \tag{3}$$

where I, J range from 1, 2, s is the coordinate along the ray, δ_{IJ} is the Kronecker delta symbol, and $v_{,IJ} = \partial^2 v/\partial q_I \partial q_J$ is the second-derivative matrix of the velocity field transverse to the ray. As an alternative, the dynamic ray equations can be written in Cartesian or other coordinate systems (Červený, 1985b; 2001), but the ray-centered coordinates are convenient for the discussion here. $Q(s)$ and $P(s)$ are solutions for the entire ray bundle in the vicinity of the central ray, and for specific initial conditions can be related to small offsets and angle changes of a linearized ray about the central ray. One of the initial applications of dynamic ray equations was to the computation of geometric spreading about a given central ray, with the ray amplitude related to $(\det Q(s))^{-1/2}$. For $Q(s_0)$ specified for a given source, then $Q(s)$ at the receiver can be computed by the dynamic ray equations.

The solution $X(s)$ of the dynamic ray equations can be written in terms of a 4×4 fundamental matrix $\pi(s, s_0)$ as

$$X(s) = \pi(s, s_0)X(s_0) = \begin{pmatrix} Q_1 & Q_2 \\ P_1 & P_2 \end{pmatrix} X(s_0) \ , \tag{4}$$

where Q_1, Q_2, P_1 and P_2 are 2×2 sub-matrices of $\pi(s, s_0)$. The fundamental matrix has the properties

$$\pi(s_0, s_0) = \begin{pmatrix} \delta_{IJ} & 0 \\ 0 & \delta_{IJ} \end{pmatrix} \quad \text{and} \quad \det[\pi(s, s_0)] = 1 \ , \tag{5}$$

where I, J range from 1, 2 and δ_{IJ} is the Kronecker delta symbol. Two primary initial conditions to the dynamic ray equations are

$$X(s_0) = \begin{pmatrix} Q(s_0) = 0 \\ P(s_0) \neq 0 \end{pmatrix}, \quad X(s_0) = \begin{pmatrix} Q(s_0) \neq 0 \\ P(s_0) = 0 \end{pmatrix} \ , \tag{6}$$

and are related to a point source with initial angle variations of linearized rays about the central ray, and to a planar source with initial displacement variations of linearized rays about the central reference ray, respectively.

Paraxial Gaussian Beams

A paraxial Gaussian beam is an asymptotic solution of a one-way parabolic wave equation in ray-centered coordinates about the central ray. The solution is similar in form to the high-frequency ray solution given in Eqn. (1), except that now $M(s)$ is complex with Im $(M(s))$ positive definite. It can be constructed from the linearized ray solution by using complex initial conditions $X(s_0)$ in Eqn. (4). Since the matrix $M(s)$ does not change if both $Q(s)$ and $P(s)$ are multiplied by a nonsingular matrix, the initial conditions to the dynamic ray equations can be written as

$$X(s_0) = \begin{pmatrix} I \\ M(s_0) \end{pmatrix} \ . \tag{7}$$

Since $\operatorname{Im} M(s_0)$ is positive-definite, the Gaussian beam solution will be a combination of the initial point source and plane wave solutions in Eqn. (6) which will exponentially decay away from the central ray. For a Gaussian beam solution, if $Q(s)$ is regular at one point along the ray, it is regular along the entire ray (ČERVENÝ, 1985b). Based on the regularity of $Q(s)$ for the Gaussian beam solution, then the amplitude factor $(\det Q(s))^{-1/2}$ will be nonsingular for all points along the ray.

In general, there will be three complex or six real parameters that are needed to specify $M(s)$ for a given point along the ray. The dynamic ray equations can then be used to find $M(s)$ at other points on the ray. For the special case of a circular beam at a given point on the ray, then $M(s)$ can be written as

$$M(s) = \operatorname{Re} M(s) + i \operatorname{Im} M(s) = \left[v^{-1} K(s) + \frac{i}{\pi L^2(s)} \right] \delta_{IJ} \ , \tag{8}$$

where v is the velocity along the ray, $K(s)$ is the beam-front curvature and $L(s)$ is the beam half-width at a frequency of 1 Hz. Therefore, the real part of $M(s)$ describes the curvature properties of the phase-front of the beam and the imaginary part of $M(s)$ describes the beam-width, where a smaller $\mathrm{Im}\, M(s)$ corresponds to a larger beamwidth. Extensions of high-frequency Gaussian beams to propagation in elastic media were described by ČERVENÝ and PŠENČÍK (1983a,b, 1984), and to anisotropic media by HANYGA (1986).

Expansion of High-frequency Wave Fields into Gaussian Beams

A high-frequency expansion of a wave field into Gaussian beams was proposed by POPOV (1981, 1982), and initially applied by POPOV *et al.* (1980), KATCHALOV and POPOV (1981) and ČERVENÝ *et al.* (1982). An early overview of the method was also given by ČERVENÝ (1981). The expansion of a time-harmonic wave field into Gaussian beams and evaluated at a position S in the medium can be written as

$$\vec{u}(S,\omega) = \iint_D \Phi(\gamma_I)\vec{u}^{GB}_{\gamma_I}(S,\omega,M(s_b))d^2\gamma \ , \tag{9}$$

where $\Phi(\gamma_I)$ is the weighting function, $\vec{u}^{GB}_{\gamma_I}$ are the beam solutions, and $M(s_b)$ are the beam parameters for a specified position s_b along the ray. The ray parameters γ_I $(I = 1,2)$ specify a given central ray along the initial wavefront, and the domain D depends on the type of source to be decomposed into beams. For an initial point source, the ray parameters γ_I can be specified by initial angles at the source, and for a planar wavefront, the γ_I can be specified by positions along the initial wave front. Since the central rays of the individual Gaussian beams need only be in the vicinity of the receiver, no two-point ray tracing is required for the method. Also by summing over Gaussian beams, Eqn. (9) naturally results in some smoothing of the wave field, and is therefore less sensitive to model parameterizations than the ray method.

The individual Gaussian beams in Eqn. (9) can be written in ray-centered coordinates (q_1, q_2, s) as

$$\vec{u}^{GB}_{\gamma_I}(S,\omega,M(s_b)) = \frac{\vec{U}^N(s)}{\det(Q(s))^{1/2}}e^{-i\omega(t-\tau(s)-\frac{1}{2}\vec{q}^T M(s)\vec{q})} \ , \tag{10}$$

where $M(s) = \mathrm{Re}\, M(s) + i\,\mathrm{Im}\, M(s)$, $\mathrm{Im}\, M(s)$ is positive-definite, $\det[Q(s)]^{-1/2} \neq 0$, and the beam parameters are given by $M(s_b)$ at a specified position along the ray. The values of $M(s)$ and $Q(s)$ are then computed using the dynamic ray equations above. $\vec{U}^N(s)$ is spreading-free ray amplitude given by $\vec{U}^R(s)\det(Q^R(s))^{1/2}$ where $\vec{U}^R(s)$ is the complete ray amplitude factor.

The weighting function for the asymptotic expansion of a wave field into Gaussian beams in Eqn. (9) can be written as

$$\Phi(\gamma_I) = \frac{\omega}{2\pi}\left[-\det\left(Q^T(s)(M(s) - M^R(s))Q^R(s)\right)\right]^{1/2}$$
$$= \frac{\omega}{2\pi}\left[-\det\left(P^T Q^R - Q^T P^R\right)\right]^{1/2} \tag{11}$$

where $Q^T(s)$ and $P^T(s)$ are the transposes of the complex 2×2 matrices $Q(s)$ and $P(s)$. The values $Q^R(s)$, $P^R(s)$ and $M^R(s)$ are the corresponding ray values for the given type of source. For example, if we specify $Q^R(s) = I$, $P^R(s) = 0$, and $M^R(s) = 0$ at the source, then the resulting weighting function is the same as that for the expansion of a plane wave into Gaussian beams as given by ČERVENÝ (1982). Similarly, the weighting function for the expansion of an initial point source into Gaussian beams is given by ČERVENÝ (1985b). The weighting function for the asymptotic expansion of an arbitrary initial wavefront into Gaussian beams can also be obtained (KLIMEŠ,1984a; ČERVENÝ, 1985a,b). Based on the properties of the solutions of the dynamic ray equations, the weighting factor $\Phi(\gamma_I)$ is an invariant and can be evaluated at any point along the ray (ČERVENÝ, 1985b).

If we specify the weighting function at the endpoint of the ray, the complete amplitude term for the integrand in Eqn. (9) can be written as

$$\Phi(\gamma_I)\frac{\vec{U}^N(s)}{\det(Q(s))^{1/2}} = \frac{\omega}{2\pi}\left|\det(Q^R(s))\right|^{1/2}\left[-\det\left(M(s) - M^R(s)\right)\right]^{1/2}\vec{U}^N(s) \tag{12}$$

where Re $[-\det(M(s) - M^R(s))]^{1/2} > 0$, and all parameters other than $M(s)$ are the corresponding ray values. The specification of $M(s)$ at the endpoint of the ray gives added phase stability to the Gaussian beam summations.

Different methods for performing the Fourier transform of the Gaussian beam summation to the time-domain have then been given by ČERVENÝ (1983, 1985a,b). A direct frequency domain approach can be written as

$$\vec{u}(S,t) = \frac{1}{\pi}Re\int_0^{\infty} F(\omega)\vec{u}(S,\omega)e^{-i\omega t}d\omega \ , \tag{13}$$

where $F(\omega)$ is the Fourier transform of source-time function $f(t)$, and $\vec{u}(S,\omega)$ is the frequency response given in Eqn. (9). This is the most convenient form for the incorporation of additional frequency-dependent effects such as causal attenuation. The incorporation of a general moment tensor source function has been described by ČERVENÝ *et al.* (1987).

A convolutional approach can also be used where the impulse response of the Gaussian beam summation is convolved with the source-time function. For a complex phase factor $\tau(q_i, s)$ of the form given in Eqn. (2), this can be written as

$$\vec{u}(S,t) = f(t)^*\frac{1}{\pi}\iint_D d\gamma^2 \mathrm{Im}\left(\left\{\frac{\Phi(\gamma_I)\vec{U}^N(s)\det(Q(s))^{-1/2}}{t - \tau(q_I, s)}\right\}\right) \ . \tag{14}$$

Finally, for particular choices of $f(t)$, elementary signals in the time domain can be asymptotically obtained. The most common choice is the Gabor wavelet which is a Gaussian weighted cosine function. This choice for $f(t)$ results in the fastest computational approach and can be written as

$$\vec{u}(S,t) = \iint_D d\gamma^2 \vec{u}_{\gamma_I}^{GW}(S,t) \ , \tag{15}$$

where $\vec{u}_{\gamma_I}^{GW}(S,t)$ are the individual Gabor wave packets which are now Gaussian in both space and time.

For numerical calculations, the Gaussian beam integrals in Eqns. (9), (14) and (15) must be discretized to form discrete summations of Gaussian beams. The error in performing this discretization has been described by ČERVENÝ (1985a) and KLIMEŠ (1986), and the minimization and bounding of the discretization error have been used as one selection criterion for the beam parameters.

Choices of Beam Parameters

The complex beam parameters $M(s)$ are commonly specified at either the source or receiver, although other choices, such as at interfaces, are possible as well. For the special case of a circular beam at a given position along the ray, then $M(s)$ can be written in terms of the wavefront curvature $K(s)$ and the beam half-width $L(s)$ as in Eqn. (8). However, after propagation, $M(s)$ will generally no longer represent a circular beam.

We first describe several common choices of beam parameters at the source point s_0. For Re $M(s_0) \to 0$ and Im $M(s_0) \to 0$ this generates large planar beams, and results in a plane-wave expansion at the source. A second choice of the beam parameters at the source for a chosen Re $M(s_0)$ is to specify Im $M(s_0)$ to produce the smallest beamwidth at the receiver. For this case, Im $M(s_0)$ can be written as

$$\text{Im } M(s_0) = C\left\{[Q_2^{-1}(s)Q_1(s) - \text{Re}\, M(s_0)]^2 + A^2\right\}^{1/2} \ , \tag{16}$$

where C is a constant, A is a constant 2×2 matrix and Q_1 and Q_2 are subcomponent matrices of the fundamental ray matrix at the receiver given in Eqn. (4) (ČERVENÝ, 1985b). Choosing Re $M(s_0) \to 0$ with $C = 1$ and $A = 0$ gives the so-called optimal beam choice for planar beams at the source that results in the smallest beams at the receiver (ČERVENÝ *et al.*, 1982).

The beam parameters can also be chosen at the receiver. For Re $M(s) \to 0$ and Im $M(s) \to 0$, this gives large planar beams at the receiver and results in Chapman-Maslov seismograms (KLIMEŠ, 1984b; ČERVENÝ, 1985b). Alternatively, for velocity gradients or topography at the receiver, then Re $M(s)$ can be specified to give effective planar beams at the receiver, providing stable summation results.

Approximate ray synthetic seismograms can be obtained by choosing Re $M(s)$ at the receiver to be equal to $M^R(s)$, and Im $M(s)$ very large to give small beamwidths at the receiver (ČERVENÝ, 1985a). For the practical summation of Gaussian beams for this case, then Im $M(s)$ needs to be chosen in relation to the ray-sampling interval (ČERVENÝ, 1985a).

Another choice at the receiver for a specified Re $M(s)$ is

$$\text{Im } M(s) = C\left\{[M^R(s) - \text{Re } M(s)]^2 + A^2\right\}^{1/2} , \tag{17}$$

where C is a constant, A is a constant 2×2 matrix, and $M^R(s)$ is related to the ray-curvature matrix. Similar to the specification in Eqn. (16) at the source, this gives narrow beams at the receiver for short ray paths and wider beams for longer ray paths. Specifying Re $M(s) \to 0$ results in planar beams at the receiver. ČERVENÝ (1985a,b) gave modifications of Re $M(s)$ to incorporate lateral heterogeneities or a curved interface at the receiver. With the specification of beam parameters at the receiver and these heterogeneity corrections, very stable results for the summation of Gaussian beams can be obtained. The choice in Eqn. (17) can also be derived by minimizing the discretization error from replacing a continuous Gaussian beam integral with a discrete summation (ČERVENÝ, 1985a).

An alternate approach to the selection of beam parameters is to choose broad Gaussian beams which also limit the discretization error of the Gaussian beam summation. For a specified Re $M(s)$ in the 2-D case, ČERVENÝ (1985a) specified Im $M(s)$ which results in broad beams at the receiver which still limits the discretization error given by KLIMEŠ (1986). ČERVENÝ (1985a) found that this choice gave very stable results for vertically inhomogeneous media.

In many cases, it was found that stable results can be obtained for a large range of beam parameters used in the Gaussian beam summation (ČERVENÝ *et al.*, 1982; NOWACK and AKI, 1984). For example, Fig. 1 shows the individual beams and the resulting Gaussian beam summation for a layer over a linear gradient. The model has a velocity of 5.6 km/s down to 15 km and then increases linearly to 8.0 km/s at 40 km. Two geometric arrivals occur at a distance range of 140 km as shown by the stationary phase points of the individual beam contributions in Figure 1. Figure 1A presents the results for the optimal beam parameter choice of ČERVENÝ *et al.* (1982) as given by Eqn. (16) with Re $M(s_0) = 0$, $C = 1$ and $A = 0$. Figure 1B displays the corresponding results for beams that are 16 times wider at the source than in Figure 1A. The resulting beam summations can be seen to be very similar for both beam parameter choices. Nevertheless, certain choices of beam parameters will give better results for a given type of source. For example, NOWACK and AKI (1984) found that wide planar beams at the source give better results for the expansion of a point source, while narrow planar beams at the source give better results for planar sources.

For structures with strong lateral velocity variations, larger values of Im $M(s)$ are required along the path in order to ensure validity of the individual beam elements.

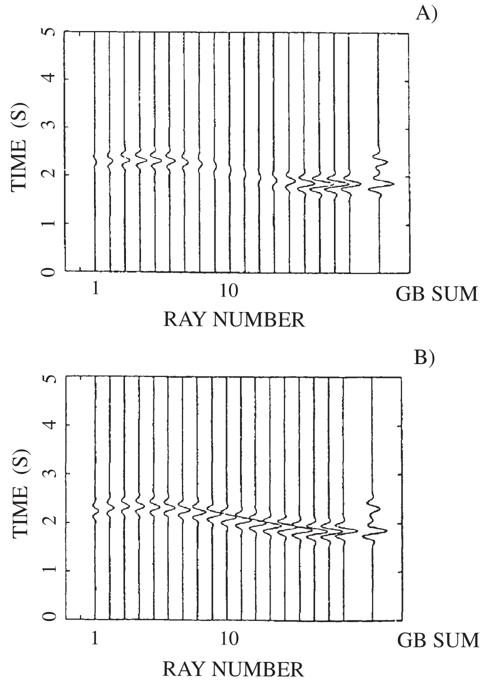

Figure 1
Gaussian beam summation for a layer over a gradient model with a 5.6 km/s layer down to 15 km and a velocity gradient from 5.6 km/s at 15 km to 8 km/s at 40 km. The individual Gaussian beam contributions are shown in the time domain using a Gabor wavelet, and the Gaussian beam summation is shown for a receiver at 140 km. A) The Gaussian beam contributions and summation are shown using the optimal beam choice of Červený *et al.* (1982). B) The Gaussian beam contributions and summation are shown for beams that are 16 times wider at the source than in A).

For example, Fig. 2 shows a random velocity layer with a thickness of 120 km. The layer is spline interpolated with velocities of 8 km/s $\pm$ 3% and a heterogeneity scale of 15 km. Gaussian beam seismograms are shown in Figure 2B for two different pulse frequencies of 1 Hz and 2 Hz, and with beam parameters specified at the source with Re $M(s_0) = 0$, $C = 1$ and $A = 0$ in Eqn. (16). These results compared well with results obtained using a parabolic finite-difference approach (Nowack and Aki, 1984). Nonetheless, smaller scale features or multi-scale structures could be more problematic for the Gaussian beam method.

Additional choices of beam parameters have been given by Madariaga (1984), Müller (1984), Weber (1988a), and Klimeš (1989b). Klimeš (1989a) describes the beam superposition in terms of Gaussian packets with a summation along the ray as well. For specific problems, additional choices of beam parameters may give more optimized results, whereas some beam parameters could lead to poor results (Nowack and Aki, 1984; Červený, 1985a; White *et al.*, 1987). This dependence on the beam parameters was also noted by Felsen (1984) and Norris (1986) and

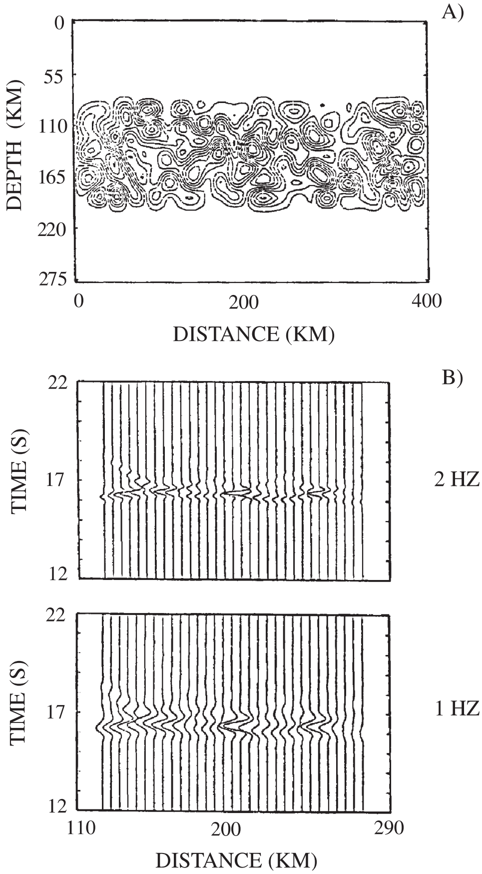

Figure 2
The Gaussian beam method applied to the propagation of a plane wave through a random velocity layer. A) Contour plot of a 120 km thick layer with randomly fluctuating velocities with velocities of 8 km/s ± 3% and a scale length of 15 km. B) Gaussian beam wave fields for a vertically incident plane wave propagated through the random velocity layer in A) for pulse center frequencies of 1 and 2 Hz (from NOWACK and AKI, 1984).

resulted in various extensions to the original method as described below. Nonetheless, the original Gaussian beam method provides very stable, nonsingular results over a wide range of beam parameters for smoothly varying media.

Applications of the Gaussian Beam Method

In this section, an overview is given of representative applications of the Gaussian beam method. In addition to applications of the method by the original Czech and Russian groups, groups at MIT led by K. Aki and later at USC, as well as groups in

France, Germany and elsewhere performed early studies of the Gaussian beam method. As an example of early work from the MIT group, NOWACK and AKI (1984) performed a number of validity tests of the method with different beam parameters. They then applied the Gaussian beam method to random, smoothly varying media (see Figure 2), as well as to focusing effects from a volcanic structure. NOWACK and AKI (1986) applied the method to the inversion of waveform data for velocity structure.

MADARIAGA (1984) developed the Gaussian beam method for vertically varying media and used modified initial conditions expressed in terms of WKB and point source solutions. Gaussian beams were also specified in geographic coordinates. MADARIAGA and PAPADIMITRIOU (1985) then used Gaussian beams for the modeling of upper mantle phases. Different beam parameter choices were investigated by MÜLLER (1984) and WEBER (1988a). WEBER (1988b) applied the method to the modeling of regional refraction data. A recent application to upper mantle phases was given by LEBORGNE *et al.* (1999).

CORMIER and SPUDICH (1984) investigated waveform complexity from focusing in the heterogeneous fault zone of the Hayward-Calaveras fault system using Gaussian beams. NOWACK and CORMIER (1985) then applied the Gaussian beam method to the 3-D structure beneath the seismic array NORSAR. CORMIER (1987) applied the method to the focusing and defocusing of incident teleseismic waves by the 3-D structure at the Nevada test site. CORMIER and SU (1994) used the Gaussian beam method to study the effects of 3-D crustal structure on the estimation of fault slip history and ground motion.

Gaussian beams were applied to surface waves by YOMOGIDA (1985, 1987), YOMOGIDA and AKI (1985) and JOBERT (1986, 1987) using vertical adiabatic modes and horizontal beams along the surface. The transformation of JOBERT and JOBERT (1983) was used to perform 2-D ray tracing on a sphere. FRIEDERICH (1989) directly propagated Gaussian beams for long-period surface waves on a sphere. YOMOGIDA and AKI (1987) used the Gaussian beam method to invert surface wave amplitude and phase data for velocity anomalies in the Pacific Ocean basin. A research group lead by K. Aki and T.L. Teng at USC performed further surface wave studies using Gaussian beams. For example, ZHENG *et al.* (1989) used surface waves to map the crust and upper mantle in the Arctic region, KATO *et al.* (1993) studied surface wave propagation in sedimentary basins in Japan, QU *et al.* (1994) applied Gaussian beams for short-period surface wave propagation in Southern California, and CHEN *et al.* (1998) studied short-period surface waves in Taiwan.

CORMIER (1989) applied the Gaussian beam method to the diffraction of seismic pulses from downgoing subducted slabs. WEBER (1990) and SEKIGUCHI (1992) then used Gaussian beams to investigate the influence on *P*-wave travel times and amplitudes of heterogeneous subduction zones. CORMIER (1995) performed time-domain modeling of PKIKP precursors for lower mantle heterogeneities. Studies of the lower mantle using Gaussian beams were also conducted by WEBER and DAVIS (1990) and WEBER (1993).

Further applications by Russian and Czech groups included KATCHALOV *et al.* (1983), GRIKUROV and POPOV (1983), KATCHALOV and POPOV (1985, 1988). Modeling in 3-D was performed by ČERVENÝ and KLIMEŠ (1984). The relation between the Gaussian beam method and the Maslov method was investigated by KLIMEŠ (1984b). ČERVENÝ *et al.* (1987) applied the Gaussian beam method to the modeling of extended earthquake sources in laterally varying structures and found good agreement with results from finite-element modeling and the isochron method. An overview of the Gaussian beam method was given by BABICH and POPOV (1989) along with additional references up to that time.

Studies of the range of validity of rays and beams were conducted by BEN-MENAHEM and BEYDOUN (1985) and BEYDOUN and BEN MENAHEM (1985). Applications of the Gaussian beam method in other fields have included that of PORTER and BUCKER (1987) who applied the method to ocean acoustics (see also, JENSEN *et al.*, 1994). The method was applied to atmospheric acoustics by GABILLET *et al.* (1992).

Perturbation methods were used to compute approximate rays and beams in complicated media from results in more simple media by FARRA and MADARIAGA (1987). NOWACK and LUTTER (1988) applied ray perturbation methods for the investigation of linearized rays and their influence on the inversion of travel times and amplitudes. NOWACK (1990) then used perturbation methods for the calculation of Gaussian beam seismograms in a laterally varying perturbed medium using results computed in a laterally homogeneous medium.

As a recent example of the Gaussian beam method, NOWACK and STACY (2002) applied the method to the calculation of interference head waves for an interface with a velocity gradient beneath. Interference waves result from multiple bounces on the underside of an interface, as well as wide-angle reflected and head waves. In an earlier study, CORMIER and RICHARDS (1977) used a full waveform technique to sum gallery phases for a gradient beneath an interface and applied this to the inner core boundary (AKI and RICHARDS, 1980). In Fig. 3, synthetic seismograms are shown using the Gaussian beam and reflectivity methods for a 6 km/s layer from 0 to 25 km over a velocity gradient going from 8 km/s at 25 km to 8.57 km/s at 40 km depth. The source depth was 6 km and the source-time function was a Gabor wavelet with a dominant frequency of 2.77 Hz.

Reflectivity synthetics for this model are displayed in Figure 3A where the first arriving phases are the direct and interference *P* wave phases from the interface at 25 km. Later phases such as the surface reflected *P* waves and *S* waves are also seen. The Gaussian beam synthetics are shown in Figure 3B and include only the direct *P* waves, interference waves and wide-angle reflections from the interface. For the interference waves produced by the gradient, the semi-automatic choice of ČERVENÝ (1985a) was used which specifies broad beams at the receiver that also limits the discretization error of the Gaussian beam summation. This choice was specified in order to produce stable results and also avoid caustics for the interference waves. To

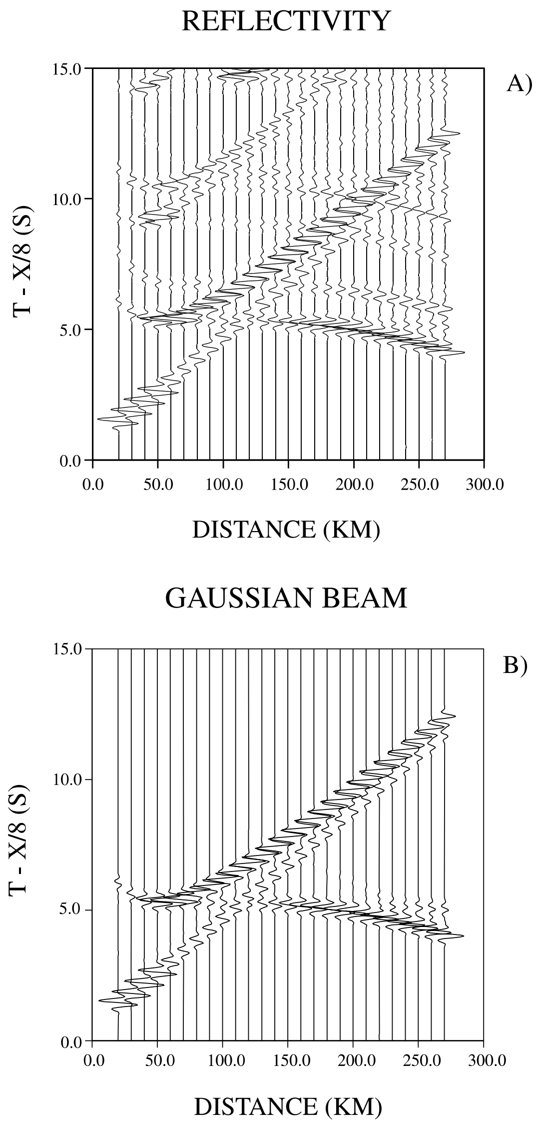

Figure 3
Synthetic seismograms are shown for a model with a layer of 6 km/s over a velocity gradient from 8 km/s at 25 km to 8.57 km/s at 40 km. Reflectivity synthetics are shown in A) and Gaussian beam synthetics are shown in B). Only the direct *P* wave and the interference and wide-angle phases from the interface at 25 km are shown for the Gaussian beam synthetics (from NOWACK and STACY, 2002).

model the wide-angle reflections somewhat broader beams were used to ensure that the direct head-wave contribution was obtained. For the direct wave, the Gaussian beam method was run in ray mode since the direct wave is standard for this case. The final Gaussian beam synthetics in Figure 3B are the summation of results for these different phases. Figure 3 shows that a good agreement between the results from the Gaussian beam and reflectivity methods was obtained for the first-arrival and wide-angle *P*-wave phases. In addition, the Gaussian synthetics were obtained for a small fraction of the computing time as the reflectivity results.

The relation between the Gaussian beam method and the use of complex source points was presented by FELSEN (1984) and WU (1985) (see also, DESCHAMPS, 1971; KELLER and STREIFER, 1971), and extensions of the Gaussian beam method using complex source points are described below. FELSEN (1984) also recommended propagation corrections for lateral beam shifts at interfaces as a way to better use narrow paraxial beams in beam summations. However, this correction was later incorporated by GAO *et al.* (1990) and was shown to be insufficient to provide the head-wave contribution when using only paraxially narrow beams in the summation.

Extensions of the Gaussian Beam Method

Although the original Gaussian beam method has been successful in modeling smoothly varying media, structures with discontinuities or corner points have presented difficulties for the method. This was identified by NOWACK and AKI (1984), KONOPASKOVA and ČERVENÝ (1984a,b), and WHITE *et al.* (1987) who demonstrated that broad beams are required to obtain the head waves at interfaces. Also, GEORGE *et al.* (1987) showed that corner points present difficulties when paraxially approximated Gaussian beams are used. For structures with strong heterogeneities, WHITE *et al.* (1987) showed that the Gaussian beam method depends on the chosen beam parameters. Numerous studies have attempted to extend the Gaussian beam method, and these have come under several categories including Gaussian beams as building blocks in other methods such as the boundary integral method, more exact beam elements and more complete beam superposition algorithms.

An expansion of a time-harmonic point source was performed by NORRIS (1986) using a distribution of exact complex source points. LU *et al.* (1987) also used complex source points to model reflections from an interface using a complex Huygen's principle, and HEYMAN (1989) extended this to time-domain point sources. Further applications of summation of complex source points are described by DEZHONG (1995) and NORRIS and HANSEN (1997). For complex sources in general heterogeneous media, complex rays must be used for ray tracing. An overview of complex rays is given by KRAVSTOV *et al.* (1999) (see also, THOMSON, 1997). As an alternative to complex ray tracing, ZHU and CHUN (1994a,b) used ray perturbation methods to obtain complex rays.

Another extension of the Gaussian beam method has been to construct superpositions of either complex source points or paraxial beam elements for extended sources. EINZIGER *et al.* (1986) used a Gabor expansion described below to study an extended aperture by the superposition of beam waves, where each of the elementary beams was specified on a phase-space lattice of shifted and tilted beams. The propagation of waves away from extended apertures by phase-space beam summations was investigated by several authors including MARCIEL and FELSEN (1989) and STEINBERG *et al.* (1991a). Self-consistent Gaussian beam superpositions using Gabor expansions were investigated by FELSEN *et al.* (1991) for 2-D applications and KLOSNER *et al.* (1992) for 3-D applications. The use of pulsed-time signals instead of time-harmonic signals for the beam summations was also considered (see for example, STEINBERG *et al.*, 1991b; MELAMED, 1997).

The Gabor expansion was initially described by GABOR (1946) and is related to windowed Fourier transforms. It can be written as

$$u(x) = \sum_n \sum_m \Phi(n,m) w(x - mL) e^{in\Omega x} \ , \tag{18}$$

where L is the sampling in position, Ω is the sampling in wavenumber, $\Phi(n,m)$ is the weighting function, and $w(x)$ is the window function. An example of a window function is the Gaussian window, $w(x) = (2^{1/2}/L)^{1/2} e^{-\pi x^2/L^2}$. The sampling parameter L also determines the width of the Gaussian window functions. The Gabor expansion was investigated by BASTIAANS (1980) who derived a set of biorthogonal expansion coefficients $\Phi(n,m)$ for a given function $u(x)$ in the case of critical sampling, where critical sampling results when the sampling in position L and the sampling in wavenumber Ω are related by $\Omega L = 2\pi$.

If the phase-shifted Gaussian window functions are considered as initial Gaussian beams in the aperture plane, then Eqn. (18) can be interpreted as a decomposition of an initial wave field into shifted and tilted Gaussian beams. Figure 4A presents an example of a 2-D phase-space lattice representing the wave field in the aperture (from FELSEN *et al.*, 1991). Figure 4B displays an example of a shifted and tilted beam element, where $\sin\theta$ is related to the wavenumber. Assuming that the initial Gaussian beams can either be exactly or asymptotically propagated away from the aperture, then the resulting wave field can be written

$$u(x,z) = \sum_n \sum_m \Phi(n,m) u_{n,m}(x,z) \ , \tag{19}$$

where $u_{n,m}(x,z)$ are the individual beams for $z > 0$ and $\Phi(n,m)$ are the weight functions. Several applications of Gabor expansions in optics have been described by BASTIAANS (1998).

Although, this approach provides a self-consistent selection criterion for the summation of Gaussian beams, it was observed by DAUBECHIES (1990) that the Gabor expansion coefficients of BASTIAANS (1980) at critical sampling are only marginally stable. Nonetheless, a useful localization can still be obtained for a Gabor expansion when oversampled frames are used, where $\Omega L < 2\pi$. Although the oversampled frames are nonunique, a minimum norm solution for the coefficients can be obtained (WEXLER and RAZ, 1990; QIAN and CHEN, 1993; ZIBULSKI and

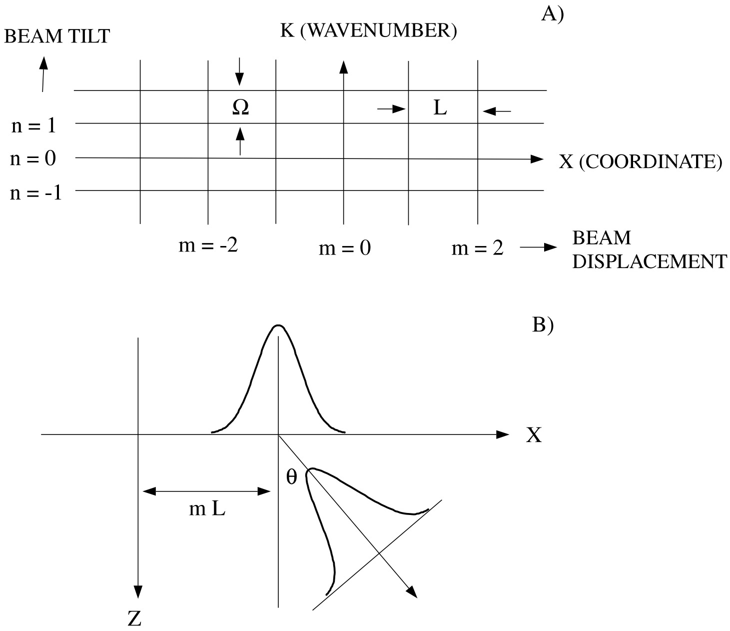

Figure 4
Summation of beams on a phase-space lattice. A) A phase-space lattice is shown for lateral beam displacement and beam tilt. The beam sampling is L and the wavenumber sampling is Ω which is related to the beam tilt. B) An example of a laterally displaced and tilted beam (from FELSEN *et al.*, 1991).

ZEEVI, 1993; BASTIAANS and GEILEN, 1996). An example of an oversampled Gabor expansion of an initial wave field propagated away from an aperture was given by LUGARA and LETROU (1998).

In geophysics, there have been several applications which have attempted to increase the spectral content of the Gaussian beam expansion. These have included the studies by WANG and WALTHAM (1995a,b), who used the stability and versatility of Gaussian beam tracking in smoothly heterogeneous media along with edge and tip diffracted waves to generalize the Gaussian beam decomposition providing more spectral content in the decomposition. Another approach has been to expand the wave field into so-called coherent states, and this method has been described by FOSTER and HUANG (1991) and THOMSON (2001) for geophysical applications based on the work of KLAUDER (1987a,b). This approach is also related to a Gaussian-windowed Fourier transform of the wave field.

BENITES and AKI (1989) used Gaussian beams as building blocks within the framework of a boundary integral method, and thus used beams in a different type of expansion of the wave field. BENITES and AKI (1994) then used this boundary integral–Gaussian beam approach for the calculation of ground motions in sedimentary basins with velocity gradients and in models with surface topography.

An important development in exploration geophysics has been the use of Gaussian beam decompositions for the imaging of seismic reflection data (COSTA, *et al.*, 1989; LAZARATOS and HARRIS, 1990; HILL, 1990). In these studies, reflection data are decomposed into local slant or beam stacks (RAZ, 1987) and matched to the seismic data at the surface. These local slant-stacks are then used within Gaussian beam migration algorithms. These approaches were found to be very successful in imaging steep dips using Gaussian beam propagation and superposition. An example of steep dip imaging of a salt dome by Gaussian beam migration was presented by HILL *et al.* (1991), and a numerical overview of the method was provided by HALE (1992a,b). Gaussian beam migration has been applied to anisotropic media by ALKHALIFAH (1995), and the use of Gaussian beam summations for prestack reflection data has been described by HILL (2001).

Conclusions

The Gaussian beam method has been shown to be a very stable asymptotic method for the computation of high-frequency wave fields in smoothly varying inhomogeneous media. One of the advantages of the method is that individual Gaussian beam components have no singularities along their paths. This assures the summation of Gaussian beams to be regular everywhere. The Gaussian beam method also introduces smoothing. Therefore, the method is not as sensitive to model parameterizations as the ray method. Another advantage is that the Gaussian beam method does not require two-point ray tracing. A number of successful applications of the method have been presented. However, the selection of beam parameters is still a topic of ongoing research and different extensions of the method have been reviewed. Nonetheless, the advantages of Gaussian beam methods will continue to make them useful for the modeling of high-frequency seismic waves in heterogeneous media.

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