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Wave Propagation, Scattering and Imaging Using Dual-domain One-way and One-return Propagators

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Abstract — Dual-domain one-way propagators implement wave propagation in heterogeneous media in mixed domains (space-wavenumber domains). One-way propagators neglect wave reverberations between heterogeneities but correctly handle the forward multiple-scattering including focusing/defocusing, diffraction, refraction and interference of waves. The algorithm shuttles between space-domain and wavenumber-domain using FFT, and the operations in the two domains are self-adaptive to the complexity of the media. The method makes the best use of the operations in each domain, resulting in efficient and accurate propagators. Due to recent progress, new versions of dual-domain methods overcame some limitations of the classical dual-domain methods (phase-screen or split-step Fourier methods) and can propagate large-angle waves quite accurately in media with strong velocity contrasts. These methods can deliver superior image quality (high resolution/high fidelity) for complex subsurface structures. One-way and one-return (De Wolf approximation) propagators can be also applied to wave-field modeling and simulations for some geophysical problems. In the article, a historical review and theoretical analysis of the Born, Rytov, and De Wolf approximations are given. A review on classical phase-screen or split-step Fourier methods is also given, followed by a summary and analysis of the new dual-domain propagators. The applications of the new propagators to seismic imaging and modeling are reviewed with several examples. For seismic imaging, the advantages and limitations of the traditional Kirchhoff migration and time-space domain finite-difference migration, when applied to 3-D complicated structures, are first analyzed. Then the special features, and applications of the new dual-domain methods are presented. Three versions of GSP (generalized screen propagators), the hybrid pseudo-screen, the wide-angle Padé-screen, and the higher-order generalized screen propagators are discussed. Recent progress also makes it possible to use the dual-domain propagators for modeling elastic reflections for complex structures and long-range propagations of crustal guided waves. Examples of 2-D and 3-D imaging and modeling using GSP methods are given.

Key words: Wave propagation, scattering, seismic imaging, modeling, one-way propagation, depth migration.

1. Introduction

Perturbation approach is one of the well-known approaches for wave propagation, scattering and imaging (see Ch. 9 of MORSE and FESHBACH, 1953; Ch. 13 of AKI and RICHARDS, 1980; WU, 1989). Traditionally, perturbation methods are used only

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for weakly inhomogeneous media and short propagation distance. However, recent progress in this direction has led to the development of iterative perturbation solutions in the form of one-way marching algorithm for scattering and imaging problems in strongly heterogeneous media. In this article, a historical review and theoretical analysis regarding the perturbation approach, including the Born, Rytov, and De Wolf approximations, are given in section 2. The relative strong and weak points of the Born and Rytov approximations are analyzed. Since the Born approximation is a weak scattering approximation, it is not suitable for large volume or long-range numerical simulations. The Rytov approximation is a smooth scattering approximation, which works well for long-range small-angle propagation problems, but is not applicable to large-angle scattering and backscattering. Then the De Wolf approximation (multiple-forescattering-single-backscattering, or ''one-return approximation'') is introduced to overcome the limitations of the Born and Rytov approximations in long-range forward propagation and backscattering calculations, which can serve as the theoretical basis of the new dual-domain propagators. A review of classical dual-domain propagators (phase-screen or split-step Fourier method) is also given, followed by a summary and analysis of the new dual-domain propagators, in section 3. The iterative perturbation approach has been developed in parallel to the operator splitting approach, and has led to the generalization of phase-screen approximation for scalar waves to the elastic wave screen propagators (see section 3.3). Three versions of GSP (generalized screen-propagators): the hybrid pseudo-screen, the wide-angle Padé-screen, and the higher-order generalized screen propagators are presented in section 3.4 as examples of the newly developed wide-angle dual-domain propagators. The applications of the new propagators to seismic imaging are reviewed in section 4. The advantages and limitations of the traditional Kirchhoff migration and time-space domain finite-difference migration, when applied to imaging of 3-D complicated structures, are first analyzed. Then the special features and applications of the new dual-domain methods are presented. Examples of 2-D and 3-D imaging (post- and pre-stack depth migrations) using synthetic data sets from the Marmousi model and SEG-EAEG salt model are given. Further progress also makes it possible to use the dual-domain propagators for modeling elastic reflections for complex structures and long-range propagation of crustal guided waves. These applications are briefly discussed in section 5. Conclusion is given in section 6.

2. From Born, Rytov to De Wolf

2.1 Born Approximation and Rytov Approximation: Their Strong and Weak Points

In recollection of my study and collaboration with Professor K. Aki, I would like to briefly digress to communicate my work on elastic Born scattering when I was a graduate student at MIT.

When we started to work on elastic Born scattering, we were not aware of GUBERNATIS *et al.*'s (1977a,b) work. We started from the basic principle and referred to MORSE and FESHBACH (1953) for our derivation. After a few months, I showed my derivation and part of the results to professor Aki. He was very delighted by the elegance of the theory and the practical implications of the results. He told me that I was very lucky to have achieved such a nice result on such a fundamental problem. But he added, "Such a neat result on this kind of fundamental problems should have been solved a long time ago. Somehow people neglected this spot and left some nice thing there. You are very lucky to pick up this stuff!" It turned out later, however that I was not as lucky as it appeared. When circulating our results to other colleagues, it was pointed out that similar results have been published in the Journal of Applied Physics (GUBERNATIS *et al.*, 1977a,b). Of course, there were differences. Gubernatis *et al.*'s results regard a uniform elastic inclusion; while ours pertain to an arbitrarily heterogeneous body, and we had nice expressions for the velocity-type and impedance-type heterogeneities. Nevertheless, the general form of elastic Born scattering was published in that paper. In the beginning, I felt embarrassed and was very disappointed, like a defeated hero. Later I recovered from that mode of debacle. I comforted myself by looking at the event from a different perspective. I looked it as a test of my ability and good fortune. I said to myself that I can attack such problems and perhaps have luck in my future work. Thus I extended my work on elastic Born scattering to more general cases and to random media, with professor Aki's assistance, and published two papers in Geophysics and Journal of Geophysical Research (WU and AKI, 1985a,b), respectively. It turned out to be a prelude to my endeavor concerning the research of seismic wave propagation and scattering.

For the sake of simplicity, we consider the scalar wave case as an example. The scalar wave equation in inhomogeneous media can be written as

$$\left(\nabla^2 + \frac{\omega^2}{c^2(\vec{r})}\right)u(\vec{r}) = 0 \ , \tag{1}$$

where ω is the circular frequency, $\vec{r}$ is the position vector, and $c(\vec{r})$ is wave velocity at $\vec{r}$. Define c_0 as the background velocity of the medium, resulting in

$$(\nabla^2 + k^2)u(\vec{r}) = -k^2\varepsilon(\vec{r})u(\vec{r}) \ , \tag{2}$$

where $k = \omega/c_0$ is the background wavenumber and

$$\varepsilon(\vec{r}) = \frac{c_0^2}{c^2(\vec{r})} - 1 \tag{3}$$

is the perturbation function (dimensionless force). Set

$$u(\vec{r}) = u^0(\vec{r}) + U(\vec{r}) \ , \tag{4}$$

where $u^0(\vec{r})$ is the unperturbed wave field or "incident wave field" (field in the homogeneous background medium), and $U(\vec{r})$ is the scattered wave field. Substitute (4) into (2) and note that $u^0(\vec{r})$ satisfies the homogeneous wave equation, resulting in

$$u(\vec{r}) = u^0(\vec{r}) + k^2 \int_V d^3\vec{r}' g(\vec{r};\vec{r}')\varepsilon(\vec{r}')u(\vec{r}') \ , \tag{5}$$

where $g(\vec{r};\vec{r}')$ is the Green's function in the background medium and the integral is over the entire volume of the medium. This is the Lippmann-Schwinger integral equation. Since the field $u(\vec{r})$ under the integral is the total field which is unknown, equation (5) is not an explicit solution but an integral equation.

Born Approximation

Approximating the total field under the integral with the incident field $u^0(\vec{r}')$, we obtain the Born Approximation

$$u(\vec{r}) = u^0(\vec{r}) + k^2 \int_V d^3\vec{r}' g(\vec{r};\vec{r}')\varepsilon(\vec{r}')u^0(\vec{r}') \tag{6}$$

In general, the Born approximation is only valid when the scattered field is much smaller than the incident field, which implies that the heterogeneities are weak and the propagation distance is short. However, the valid regions of Born approximation are very different for forward scattering and for backscattering. Forward-scattering divergence or catastrophe is the weakest point of Born approximation. For simplicity, we will use "forescattering" to stand for "forward scattering." As can be seen from (6), the total scattering field is the sum of scattered fields from all parts of the scattering volume. Each contribution is independent from other contributions since the incident field is not updated by the scattering process. In the forward direction, the scattered fields from each part propagate with the same speed as the incident field, so they will be coherently superposed, leading to the linear increase of the total field. The Born approximation has no energy conservation. The energy increase will be fastest in the forward direction, resulting in a catastrophic divergence for long distance propagation. This can be demonstrated schematically as shown in Figure 1a. The medium is divided into blocks each represented by a concentrated scatterer at its center. It also can be considered as a discrete scattering medium. At the observation point, the total field will be the sum of the incident field (without any correction) and the scattered fields from all parts of the scattering volume. On the contrary, backscattering behaves quite differently from forescattering. As shown in Figure 1b, since there is no incident wave in the backward direction, the total observed field is the sum of all the backscattered fields from all the scatterers. However, the size of coherent stacking for backscattered waves is about $\lambda/4$ because of the two-way travel-time difference. Beyond this coherent region, all other contributions will be cancelled out. For this reason, backscattering does not have the

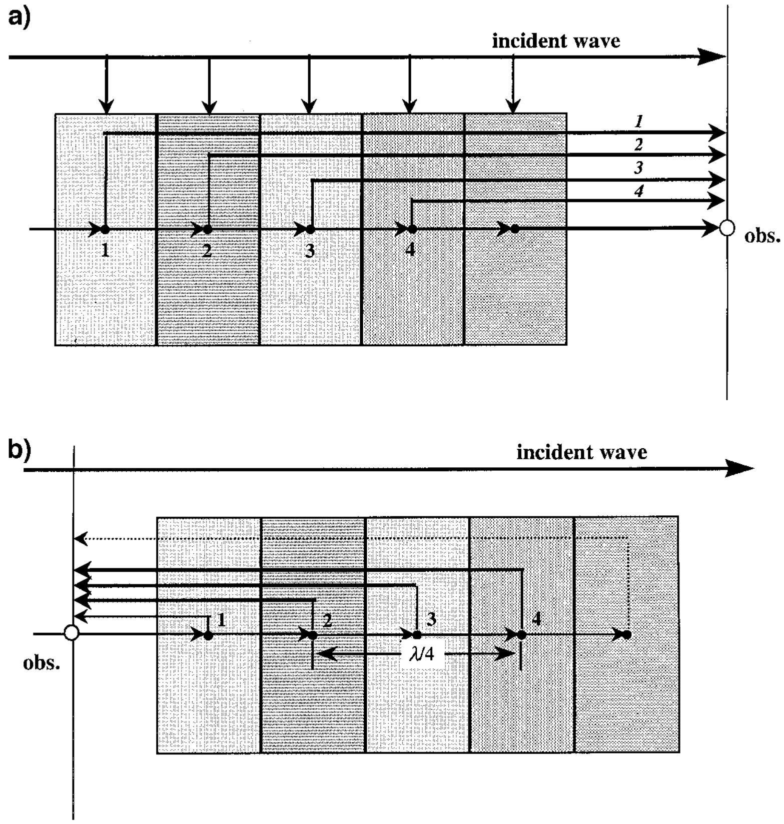

Figure 1
(a) Schematic demonstration of the forward scattering catastrophe of the Born approximation. (b) Schematic demonstration of the size of the coherent response for backscattering.

catastrophic divergence even when Born approximation is used. This can be further explained with the spectral responses of heterogeneities to scatterings with different scattering angles.

From the analysis of scattering characteristics, we know that the forescattering is controlled by the d.c. component of the medium spectrum $W(0)$, but the backscattering is determined by the spectral component at spatial frequency $2k$, i.e., $W(2k)$, where k is wavenumber of the wave field in the background medium (see, Wu and Aki, 1985b; Wu, 1989a). The d.c. component of the medium spectrum linearly increases with the propagation distance in general, while $W(2k)$ is usually considerably smaller and increases much slower than $W(0)$. The validity condition for the Born approximation is the smallness of the scattered field compared with the incident field. Therefore the region of validity of the Born approximation for backscattering is much larger than that for forescattering. The other difference between backscattering and forescattering is their responses to different types of heterogeneities. The backscattering is sensitive to the impedance type of heterogeneities, while forescattering mainly responds to the velocity type of heterogeneities. Velocity perturbation will produce travel time or phase change, which can accumulate to quite large values, causing the breakdown of the Born approximation. This kind of phase-change accumulation can be easily handled by the Rytov transformation. This is why the Rytov approximation has decidedly better performance than the Born approximation for forescattering and has been widely used for long distance propagation with only forescattering or small-angle scattering involved, such as the line-of-sight propagation of optical or radio waves (CHERNOV, 1960; TATARSKII, 1971; ISHIMARU, 1978), transmission fluctuations of seismic waves at arrays (AKI, 1973; FLATTÉ and WU, 1988; WU and FLATTÉ, 1990), diffraction tomography (DEVANEY, 1982, 1984; WU and TOKSÖZ, 1987), and seismic imaging using one-way propagators (HUANG *et al.*, 1999a,b).

Rytov Approximation

Let $u^0(\vec{r})$ be the solution in the absence of perturbations, i.e.,

$$(\nabla^2 + k^2)u^0 = 0 \tag{7}$$

and the perturbed wave field after interaction with the heterogeneity as $u(\vec{r})$. We normalize $u(\vec{r})$ by the unperturbed field $u^0(\vec{r})$ and express the perturbation of the field by a complex phase perturbation function $\psi(\vec{r})$, i.e.,

$$u(\vec{r})/u^0(\vec{r}) = e^{\psi(\vec{r})} \ . \tag{8}$$

This is the Rytov Transformation (see TATARSKII, 1971; or ISHIMARU, 1978, Ch. 17, p. 349). $\psi(\vec{r})$ denotes the phase- and log-amplitude deviations from the incident field:

$$\psi = \log u - \log u^0 = \log\left[\frac{A}{A^0}\right] + i(\phi - \phi^0) \ , \tag{9}$$

where A is the amplitude and ϕ is phase angle. Combining (2), (7) and (8) yields

$$2\nabla u^0 \cdot \nabla\psi + u^0\nabla^2\psi = -u^0(\nabla\psi \cdot \nabla\psi + k^2\varepsilon) \tag{10}$$

The simple identity

$$\nabla^2(u^0\psi) = \psi\nabla^2 u^0 + 2\nabla u^0 \cdot \nabla\psi + u^0\nabla^2\psi$$

together with (7) results in

$$2\nabla u^0 \cdot \nabla\psi + u^0\nabla^2\psi = (\nabla^2 + k^2)(u^0\psi) \ . \tag{11}$$

From (10) and (11) we obtain

$$(\nabla^2 + k^2)(u^0\psi) = -u^0(\nabla\psi \cdot \nabla\psi + k^2\varepsilon) \ . \tag{12}$$

The solution of (12) can be expressed as an integral equation:

$$u^0(\vec{r})\psi(\vec{r}) = \int_V d^3\vec{r}'g(\vec{r};\vec{r}')u^0(\vec{r}')\left[\nabla\psi(\vec{r}')\cdot\nabla\psi(\vec{r}') + k^2\varepsilon(\vec{r}')\right], \tag{13}$$

where $g(\vec{r};\vec{r}')$ is the Green's function for the background medium, $u^0(\vec{r}')$ and $u^0(\vec{r})$ are the incident field at $\vec{r}'$ and $\vec{r}$, respectively.

Equation (13) is a nonlinear (Ricatti) equation. Assuming $|\nabla\psi\cdot\nabla\psi|$ is small with respect to $k^2|\varepsilon|$, we can neglect the term $\nabla\psi\cdot\nabla\psi$ and obtain a solution known as the Rytov approximation:

$$\psi(\vec{r}) = \frac{k^2}{u^0(\vec{r})}\int_V d^3\vec{r}'g(\vec{r};\vec{r}')\varepsilon(\vec{r}')u^0(\vec{r}'). \tag{14}$$

Now we discuss the relation between the Rytov and Born approximations, and their strong and weak points, respectively. By expanding e^{ψ} into power series, the scattered field can be written as

$$u - u^0 = u^0(e^{\psi} - 1) = u^0\psi + \tfrac{1}{2}u^0\psi^2 + \cdots, \tag{15}$$

When $\psi \ll 1$, i.e., the accumulated phase change is less than one radian (corresponding to about one sixth of the wave period), the terms of ψ^2 and higher terms can be neglected, and

$$u - u^0 = u^0\psi = k^2\int_V d^3\vec{r}'g(\vec{r};\vec{r}')\varepsilon(\vec{r}')u^0(\vec{r}') \tag{16}$$

which is the Born approximation. This indicates that when $\psi \ll 1$, Rytov approximation reduces to Born approximation. In case of large phase-change accumulation, for which Born approximation is no longer valid, Rytov approximation still holds as long as the condition $|\nabla\psi\cdot\nabla\psi| \ll k^2|\varepsilon|$ is satisfied.

Let us look at the implication of the condition $|\nabla\psi\cdot\nabla\psi| \ll k^2|\varepsilon|$ for the Rytov approximation. Assume that the observed total field after the wave interacted with the heterogeneities is nearly a plane wave:

$$u = Ae^{i\vec{k}\cdot\vec{r}}$$

which could be the refracted wave in the forward direction, or the backscattered field. Since the incident wave is

$$u^0 = A_0e^{i\vec{k}_0\cdot\vec{r}}$$

the complex phase field ψ can be written as

$$\psi = \log(A/A_0) + i(\vec{k} - \vec{k}_0)\cdot\vec{r} \tag{17}$$

and

$$\nabla\psi = \nabla\log(A/A_0) + i(\vec{k} - \vec{k}_0) \tag{18}$$

$$\nabla\psi\cdot\nabla\psi = |\nabla\log(A/A_0)|^2 - \left|\vec{k} - \vec{k}_0\right|^2 + 2i(\vec{k} - \vec{k}_0)\cdot\nabla\log(A/A_0) \ . \tag{19}$$

Normally wave amplitudes vary much slower than the phases, so the major contribution to $\nabla\psi\cdot\nabla\psi$ in (19) is from the phase term $\left|\vec{k} - \vec{k}_0\right|^2$. Therefore, the condition for the Rytov approximation can be approximately stated as

$$\left|\vec{k} - \vec{k}_0\right|^2 = 4k_0^2\sin^2\frac{\theta}{2} \ll k_0^2|\varepsilon| \tag{20}$$

where θ is the scattering angle. Therefore the Rytov approximation is only valid when the scattering angle (deflection angle) is small enough to satisfy

$$\sin\frac{\theta}{2} \ll \sqrt{\frac{1}{4}\varepsilon} = \frac{1}{2}\sqrt{\frac{c_0^2 - c^2(\vec{r})}{c^2(\vec{r})}} \ . \tag{21}$$

This is a point-to-point analysis for the contributions from different terms (for example, the terms in the differential equation (12)). For the integral equation (13), one needs to estimate the integral effects of $\nabla\psi\cdot\nabla\psi$ and $k^2\varepsilon$. The heterogeneities need to be smooth enough to guarantee the smallness of the integral of $\nabla\psi\cdot\nabla\psi$ which is related to scattering angles, in comparison with the total scattering contribution $k^2\varepsilon$. Regardless, the Rytov approximation is totally inappropriate for backscattering. In the exactly backward direction, $\theta = 180°$ and $\sin\theta/2 = 1$, inequality (21) is hardly to be satisfied. Therefore, although not explicitly specified, the Rytov approximation is a somewhat small-angle approximation. Together with the parabolic approximation, they formed a set of analytical tools widely used for the forward propagation and scattering problems, such as the line-of-sight propagation problem (e.g., FLATTÉ, 1979; ISHIMARU, 1978; TATARSKII, 1971). The Rytov approximation is also used in modeling transmission fluctuation for seismic array data (WU and FLATTÉ, 1990), diffraction tomography (DEVANEY, 1982, 1984; WU and TOKSÖZ, 1987). TATARSKII (1971, Ch. 3B) discussed the relation of the Rytov approximation and parabolic approximation.

2.2 De Wolf Approximation

We see the limitations of both the Born and Rytov approximations. Even in weakly inhomogeneous media we need better tools for wave modeling and imaging for long distance propagation. Higher order terms of the Born series (defined later in this section) may help in some cases. However, for strong scattering media, Born series will either converge very slow, or become divergent. That is because the Born series is a global interaction series, each term of which is global in nature. The first term (the Born approximation) is a global response and the higher terms are just global corrections. If it makes an undue error in the first step, it will be hard to correct later. One solution to the divergence of the scattering series is the renormalization procedure. Renormalization methods try to split the operations so that the scattering series can be reordered into many sub-series. We hope some sub-series can be summed up theoretically so that the divergent elements of the series can be removed. De Wolf approximation splits the scattering potential into forescattering and backscattering parts and renormalizes the incident field and Green's function into the forward propagated field and forward propagated Green's function (forward propagator), respectively (DE WOLF, 1971, 1985). The forward propagated field u_f is the sum of an infinite sub-series which includes all the multiply forescattered fields. The forward propagator G_f is the sum of a similar sub-series which includes multiple forescattering corrections to the Green's function. The De Wolf approximation is also called "one-return approximation" (WU, 1996; WU and HUANG, 1995; WU *et al.*, 2000a,b), since it is a multiple-forescattering-single-backscattering (MFSB) approximation. It is also somewhat of a local Born approximation with both the incident field and Green's function (propagator) calculated by one-way forward propagators. From previous sections we know that Born approximation works well for backscattering locally. With the renormalized incident field and Green's function the local Born (MFSB) proved to work surprisingly well for many practical applications. The key is to have good forward propagators. RINO (1988) has obtained better approximation than MFSB in the wavenumber domain and pointed out the error of De Wolf approximation in the calculation of backscattering enhancement. The error (overestimation) is again due to the violation of energy conservation law by the Born approximation. Even with forescattering correction, the backscattered energy is still not removed from the forward propagated waves for the local Born approximation. However, for short propagation distances in exploration seismology, the errors in reflection amplitudes may not become a serious problem.

In the appendix, we give a brief derivation of De Wolf approximation using formal operator algebra (DE WOLF, 1985). In this section, we will adopt an intuitive approach of derivation to discerns the physical meaning of the approximation. De Wolf approximation bears similarity to the Twersky approximation for discrete scatterers (TWERSKY, 1964; ISHIMARU, 1978). The Twersky approximation includes all the multiple scattering except the reverberations between pairs of scatterers, which excludes the paths which connect the two neighboring scatterers more than once. The Twersky approximation has less restrictions and therefore a wider range of applications than the De Wolf approximation. The latter needs to define the split of forward and back scatterings. We define the scattering to the forward hemisphere as forescattering and its complement as backscattering.

The Lippmann-Schwinger equation (5) can be written symbolically as

$$u = u^0 + G_0 \varepsilon u \quad , \tag{22}$$

where ε is a diagonal operator in space domain, and G_0 is a nondiagonal integral operator. If the reference medium is homogeneous, G_0 will be the volume integral with the Green's function $g_0(\vec{r};\vec{r}')$ as the kernel. Formally (22) can be expanded into an infinite scattering series (Born series)

$$u = u^0 + G_0 \varepsilon u^0 + G_0 \varepsilon G_0 \varepsilon u^0 + \cdots \quad . \tag{23}$$

If we split the scattering potential into the forescattering and backscattering parts

$$\varepsilon = \varepsilon_f + \varepsilon_b \tag{24}$$

and substitute it into (23), we can have all combinations of multiple forescattering and backscattering. We neglect the multiple backscattering (reverberations), i.e., drop all the terms containing two or more backscattering potentials, resulting in a multiple scattering series which contains terms with only one ε_b.

The general term will look like

$$G_0 \varepsilon_f G_0 \varepsilon_f \cdots G_0 \varepsilon_b G_0 \varepsilon_f \cdots G_0 \varepsilon_f u^0 \quad . \tag{25}$$

The multiple forescattering on the left side of ε_b can be written as

$$G_f^m = [G_0 \varepsilon_f]^m G_0 \tag{26}$$

and on its right side,

$$u_f^n = [G_0 \varepsilon_f]^n u^0 \quad . \tag{27}$$

Collecting all the terms of G_f^m and u_f^n respectively, we have

$$\begin{aligned} G_f^M &= \sum_{m=0}^{M} [G_0 \varepsilon_f]^m G_0 \\ u_f^N &= \sum_{n=0}^{N} [G_0 \varepsilon_f]^n u^0 \quad . \end{aligned} \tag{28}$$

Let M and N go to infinite, then the renormalized G_f (forward propagator) and u_f (forescattering corrected incident field) are:

$$\begin{aligned} G_f &= \sum_{m=0}^{\infty} [G_0 \varepsilon_f]^m G_0 \\ u_f &= \sum_{n=0}^{\infty} [G_0 \varepsilon_f]^n u^0 \end{aligned} \tag{29}$$

and De Wolf approximation becomes

$$u = u_f + G_f \varepsilon_b u_f \quad . \tag{30}$$

The observed total field u in (30) is different for different observation geometries. For transmission problems, the backscattering potential has no effect under the De Wolf approximation,

$$u_{\text{transmission}} = u_f \quad . \tag{31}$$

On the other hand, for reflection measurement, that is, when the observations are at the same level as or behind the source with respect to the propagation direction, there is no u_f in the total field (30),

$$u_{\text{reflection}} = G_f \varepsilon_b u_f \quad . \tag{32}$$

Write it into integral form, (30) becomes

$$u(\vec{r}) = u_f(\vec{r}) + \int_V d^3\vec{r}' g_f(\vec{r},\vec{r}')\varepsilon_b(\vec{r}')u_f(\vec{r}') \quad . \tag{33}$$

Note that both the incident field and the Green's function have been renormalized by the multiple forescattering process through the multiple interactions with the forward-scattering potential ε_f.

3. Dual-domain One-way Propagators for Scalar, Acoustic and Elastic Waves

3.1 Classical Scalar-wave Dual-domain Propagators

The first use of a dual-domain propagator can be traced back to CHANDRASEKHAR's work (1952) on the calculations of amplitude fluctuations of light (scintillations) passing through the atmosphere using the thin-phase-screen. The field fluctuation outside the extended medium containing irregularities in refractive index, can be calculated as if produced by a thin phase-changing screen (CHANDRASEKHAR, 1952; BRAMLEY, 1954, 1977). The early use of phase-screen is a single screen for the whole inhomogeneous layer (atmosphere for light or ionosphere for radio waves). The interaction with heterogeneities is concentrated at the screen: a phase-changing operation in the space domain; the propagation is in the wavenumber domain. The formulation is simple with an exponential transformation and is similar to the Rytov transformation. However, in the phase-screen approximation, the phase-function is real, not complex as in the Rytov transformation. Therefore the phase-screen only imposes phase modulation to waves passing through it. Although simple, it has advantages over the formulation using Born approximation (BOOKER and GORDON, 1950), since for strong fluctuation the accumulated phase error by Born approximation may become significant. The approach also has been used for wave propagation through a single turning point (MERCIER, 1962; SALPETER, 1967; FLATTÉ, 1979, ch. 11) and for strong fluctuation theory (see e.g., ISHIMARU, 1978, ch. 20). Later the method was extended to multi-screen to accommodate long-range propagation (HERMANN and BRADLEY, 1971; BROWN, 1973; FLECK *et al.*, 1976; FEIT and FLECK, 1978; RINO, 1978, 1982; KNEPP, 1983; MARTIN and FLATTÉ, 1988). The interaction between the heterogeneities and wavefield is through phase screens at each step along the propagation path. It is widely used for laser propagation through the atmosphere and later through optical fibers. Random media are modeled through a series of random phase-screens (ibid).

The method was introduced to ocean acoustics by HARDIN and TAPPERT (1973), TAPPERT (1974), FLATTÉ and TAPPERT (1975), and MCDANIEL (1975), and was called split-step Fourier method as a purely numerical method for solving parabolic wave equations.

The original phase-screen propagator is derived from the parabolic wave equation, and the free propagator suffered the parabolic approximation. A "wide-angle" split-step propagator has been obtained, based on the symmetric splitting of the square-root operator (FLECK *et al.*, 1976; FEIT and FLECK, 1978; THOMSON and CHAPMAN, 1983), in which the free propagator is an accurate one. The accuracy of this improved one-way propagator has been analyzed in those papers and more recently by HUANG and FEHLER (1998). The other approach to improve the phase-screen propagator was to match its travel time with the ray equation (TOLSTOY *et al.*, 1985; BERMAN *et al.*, 1989). BERMAN *et al.* (1989) changed the phase correction term of the screen into $\log n$, where n is the refraction index of the medium. However, all the improvement is kept in the realm of classical phase-screen correction.

Dual-domain one-way propagation methods were introduced to exploration seismology early in the 90s, with methods such as the split-step Fourier method (STOFFA *et al.*, 1990; LEE *et al.*, 1991), or the phase-screen method (WU and HUANG, 1992; LIU and WU, 1994) as alternatives to the time-space finite-difference solutions. These methods operate in the frequency domain and use the dual-domain implementation with operations shuttling between space and wavenumber domains by Fast Fourier Transform. Free propagation is accomplished in the wavenumber domain through a homogeneous medium which has some reference velocity. This reference velocity can vary with depth. Wave-medium interaction is done in the space domain that accounts for the effects of the heterogeneity to the wavefront. These methods have no grid dispersion and are unconditionally stable.

3.2 Wide-angle Dual-domain Propagators

The abovementioned phase-screen or split-step methods can be viewed as classical dual-domain propagator methods. These methods are accurate only for small-angle waves and cannot correctly handle large-angle waves. This severely limits its practical applications. Recently, significant progress has been made in improving the large-angle accuracy of the dual-domain methods and in extending them to acoustic and elastic waves (WU, 1994, 1996; WU and XIE, 1994; RISTOW and RUHL, 1994; WU and HUANG, 1995; HUANG and WU, 1996; HUANG and FEHLER, 1998, 2000; WU and JIN, 1997; XIE and WU, 1998, 1999, 2000; JIN and WU, 1999a,b; JIN *et al.*, 1998, 1999, 2000; HUANG *et al.*, 1999a,b; DE HOOP *et al.*, 2000; LE ROUSSEAU and DE HOOP, 2000). Various modifications and extensions have been introduced to improve the wide-angle response of the dual-domain propagators with different names for the propagators. Approximate propagators based on the use of local Born and local Rytov approximations (local Born and local Rytov propagators) have been developed by HUANG *et al.*, (1999a,b) and HUANG and FEHLER (2000a) (see also the early work of WU and HUANG, 1995). Fourier finite-difference methods developed by RISTOW and RUHL (1994), HUANG and FEHLER (2000b) and other authors are hybrid methods, which adopt the finite-difference calculations for wide-angle corrections to the phase-screen propagators. Generalized screen methods including pseudo-screen, complex screen, windowed screen, Padé-screen, and higher-order generalized screens have been developed and applied to synthetic and field data (WU, 1994; WU and HUANG, 1995; WU and JIN, 1997; JIN and WU, 1999a,b; JIN *et al.*, 1999, 2000; XIE and WU, 1998, 1999, 2000; DE HOOP *et al.*, 2000; LE ROUSSEAU and DE HOOP, 2000).

Based on the De Wolf approximation, WU (1994) derived an elastic one-way propagator, the complex-screen propagator. In the limiting case (null shear rigidity) he derived a new one-way propagator for scalar waves (WU, 1994, §5). For small angles it reduces to the classical phase-screen (“wide-angle” of FEIT and FLECK, 1978). But for large angles it keeps the form of the local Born approximation, which has better accuracy than the phase-screen solution. The new propagator is named “generalized phase-screen propagator”. Along this direction, DE HOOP *et al.*, (2000) formulated a new type of acoustic one-way propagator based on the Hamilton path-integral and pseudo-differential operator theory. Wu’s new propagator coincides with the first-order expansion of the new class of propagators identified as generalized screen propagators (GSP). The first-order approximation “generalized phase-screen propagator” was renamed as “pseudo-screen propagator” (see next section). The original form of pseudo-screen propagator has a singularity in the wavenumber domain and numerical instability. The problems were solved by the introduction of the Taylor expansion around the singularity in the extended local Born Fourier method and by the use of the Rytov approximation in the extended local Rytov Fourier method (HUANG *et al.*, 1999a,b). JIN *et al.*, (1998, 1999) solved the problems by using the Padé expansion and implementing the wide-angle corrections with an implicit finite-difference algorithm. For further development of the new propagators see the examples in the next section.

3.3 Acoustic and Elastic Screen Propagators

Dual-domain methods have also been developed for modeling elastic wave propagation in heterogeneous media (WU, 1994, 1996; WILD and HUDSON, 1998; WILD *et al.*, 2000) and for modeling primary reflections (XIE and WU, 1995, 1999, 2000; WU, 1996; WILD and HUDSON, 1998; WU and WU, 1998, 1999). Methods for wave propagation using dual-domain propagators for regional seismic waves in complicated Crustal structures (half-space screen propagators) have been developed and tested by comparing the results to finite difference solutions (WU *et al.*, 2000a,b). Dual-domain propagators for modeling acoustic wave reflections were developed in similar time (WU and HUANG, 1995; WU *et al.*, 1995; DE HOOP *et al.*, 2000).

3.4 Examples of Wide-angle Dual-domain Propagators

As we have discussed, there are many different versions of dual-domain propagators (DDP). Here we examine some generalized screen propagators (GSP) as examples. Different versions of GSP with various approximations can be derived through different approaches. The early derivation used the local Born approximation and the De Wolf approximation (WU, 1994, 1996). Later the one-way wave propagation with GSP was more rigorously cast into a Hamilton (phase space) path-integral formulation (DE HOOP *et al.*, 2000), which forms a mathematical basis for accuracy analysis and further development of screen propagators. However, for the path integral in exact form, the vertical slowness symbol is hard to solve and the implementation would be very involved even if we could find the exact form. Therefore different approximations must be invoked for practical use of the method. In the following, we discuss three versions of GSP from the viewpoint of path-integral formulation and the approximation of vertical slowness symbol: pseudo-screen approach, generalized screen series expansion, and Padé expansion approach.

Pseudo-screen propagator starts with the weak scattering assumption, so that the vertical slowness symbol can be decomposed into background and perturbation parts. The perturbation part can be derived with a local Born approximation. However, the Born approximation is basically a low-frequency approximation, and has severe phase errors for strong contrast and high-frequencies, especially for large-angle waves. In order to have better phase accuracy, which is important for imaging (migration), some high-frequency asymptotic phase-matching has been applied to the local Born solution. Even zero-order matching leads to a solution better than the classic phase-screen method (spit-step Fourier method). The term ‘‘pseudo-screen’’ first appeared in WU and DE HOOP (1996) and HUANG and WU (1996) to distinguish the new form of screen propagator from the classic phase-screen propagator. Phase-screen has operations only in the space domain so that the phase correction is accurate only for small-angle waves; while pseudo-screen has operations in both the space and wavenumber domains to improve the accuracy for large-angle waves. The operations of pseudo-screen for heterogeneity correction have deviated from the function of a physical ‘‘screen’’; and the phase-delay is angle-dependent. That is why the correction is termed ‘‘pseudo-screen.’’ The asymptotic phase-matching method used by JIN *et al.*, (1998, 1999) in the hybrid pseudo-screen propagator with a wavenumber filter in the form of continued fraction expansion can improve the large-angle wave response significantly. In the method, the wide-angle correction is implemented with an implicit finite-difference scheme and the expansion coefficients are optimized by phase-matching.

Generalized screen series expansion expands the perturbation part of the vertical slowness symbol into a series (DE HOOP *et al.*, 2000) in terms of both the smallness parameter and the smoothness parameter of the perturbations. The first-order term in smallness is in parallel with the local Born solution. Higher-order terms can improve the wide-angle performance but involve more calculations. LE ROUSSEAU *et al.*, (2000) extended the scalar GSP to transversely isotropic media with a vertical symmetry axis.

Wide-angle Padé-screen propagator starts with a smooth approximation for the vertical slowness symbol in the very beginning. The approach does not require the weak perturbation assumption and therefore can handle strong contrast media more naturally. However, the approximation applied corresponds to the local homogeneity approximation in the traditional way of expanding the square-root operator. Large errors may exist around sharp boundaries. The expansion of the symbol in terms of *h–f* series can be found in FISHMAN and MCCOY (1984). Retaining only the leading term leads to a simple form of $\gamma_{(0)}^{HF} = \{\alpha^2(X_T) - \alpha_T^2\}^{1/2}$ which is the principal part of the symbol (DE HOOP *et al.*, 2000), where γ is the vertical slowness, $\alpha(X_T)$ is the local slowness (inverse velocity) at a transverse position X_T and α_T is the horizontal slowness. After the smooth approximation (*h–f* asymptotics), the vertical slowness symbol is expanded into a Padé series and a finite-difference (FD) scheme is used to implement the wide-angle corrections (XIE and WU, 1998; XIE *et al.*, 2000).

As we pointed out, the FD implementation of wide-angle corrections in this approach makes the method resembling the Fourier finite-difference method (RISTOW and HUHL, 1994; HUANG and FEHLER, 2000b). They are both based on the local homogeneity approximation. In HUANG and FEHLER (2000b) the coefficients of the first Padé expansion are globally optimized.

4. Imaging Using Dual-domain Propagators

Two milestones in the development of seismic imaging were the introduction of the one-way (parabolic) wave equation finite-difference algorithm (CLEARBOUT, 1970, 1976) and the introduction of the Kirchhoff integral method (SCHNEIDER, 1978). Both approaches have been widely used in the industry. The application of the dual-domain technique emerged in exploration seismology only in the beginning of the 90s and is relatively new to the industry. I will give first a brief overview of the abovementioned two approaches and their limitations, followed by a summary of the features of the newly developed dual-domain propagators.

4.1 Limitations of the Kirchhoff Migration Method

The widely used Ray-Kirchhoff imaging (depth migration) method (or simply "Kirchhoff migration") is a ray-theory based method. The method uses the Kirchhoff integral with a ray-theory approximated Green function. The process consists of ray-tracing from both the source and receiver down to the imaging point and then the pickup and stack of the wavelets from all the seismic traces according to the corresponding travel times. This approach has been successfully used in areas with relatively simple structures. However, it has two fundamental limitations which cause problems in applications to complicated regions, especially in 3-D cases.

One limitation of the Kirchhoff method is its high-frequency approximation (ray approximation) of the Green's function. The Fresnel radius of a ray can be viewed as more or less the distance from the center of the ray within which the wave field has less than 180 degrees of phase difference. The Fresnel radius increases with propagation distance and hence increases with depth leading to low resolution (lateral resolution) and poor image quality for deep targets in complex region. In contrast, wave equation migration methods are based on the wave theory which includes all the frequency-dependent properties of the wave field. The decrease in resolution and image quality of wave-theory based methods with depth is significantly less severe than that of ray-theory based methods.

The other limitation of the Kirchhoff method is the low fidelity of the amplitude information carried in the imaging process and for the final image. It is difficult to obtain amplitude information for rays propagating through complex structures because of the presence of ray caustics, multiple arrivals, and interference. Wave equation migration methods maintain the true amplitude information and thus provide high-fidelity images.

Other problems with the ray-theory based Kirchhoff method include the difficulties in dealing with multiple arrival interference, caustics, chaotic rays, sensitivity to velocity structures especially those with irregular sharp interfaces, and the calculation and storage of large travel-time tables for 3-D imaging.

The fundamental limitations of the Kirchhoff method severely limit its applications for the high resolution/high fidelity imaging in complicated regions. Nevertheless it will remain to be quite useful and convenient for some industrial applications because of its flexibility in target-oriented imaging and straightforward implementation.

4.2 Space-domain One-way Wave Finite-difference Migration

The time-space (t-x) domain finite-difference algorithm is a one-way wave-equation based method that has many advantages over the ray-Kirchhoff method. Since the introduction of the method by Clearbout by the end of 60s and the beginning of 70s (CLAERBOUT, 1970, 1976, 1985) many improvements have been made in various aspects. The method keeps the basic features of wave-theory based imaging, and overcomes the fundamental limitations of ray-theory based methods. In addition, it has abundantly faster speed of computation compared to full-wave equation methods.

Despite its success, the *t-x* domain finite difference approach also has certain intrinsic difficulties, especially for 3-D imaging. One is the grid dispersion problem which originates from the rectangular discretization of space that results in different propagation speeds for waves of different angles. This dispersion will cause errors and artifacts for imaging. Suppression of these artifacts usually leads to severe attenuation of large-angle waves which are important for imaging steep structures. Other problems with the *t-x* finite-difference approach include the numerical anisotropy for 3-D geometry and difficulty in formulating a midpoint-offset domain approach for more efficient migration. Because of these problems, the Kirchhoff method has dominated the exploration industry for a long period despite its fundamental limitations.

4.3 Features of Dual-domain Propagators (DDP)

The dual-domain propagators, which are wide-angle one-way propagators, neglect wave reverberations between heterogeneities but correctly handle the forward multiple-scattering including focusing/defocusing, diffraction, refraction and interference of waves. Due to recent progress, new versions of dual-domain methods can propagate large-angle waves quite accurately in strong contrast media, resulting in superior image quality for complex geological regions.

Dual-domain methods are self-adaptive to the complexity of the medium. In homogeneous regions, the algorithm will automatically perform wavenumber domain operations that are accurate up to 90°; while in heterogeneous regions properly weighted space-domain operations will be added according to the strength of the heterogeneities. The adaptive phase-space (dual-domain) manipulation makes the best use of the operations of each domain, resulting in efficient and accurate propagators.

The wide-angle capability of these new propagators can be seen from the propagating wave fronts in strongly perturbed media. Figure 2 exhibits comparisons of wavefronts calculated using different propagators. The reference velocity used in calculating wave propagation in each case was chosen to be a factor of two different from the real velocity, so we can investigate how well the propagators correct for the difference between the chosen reference velocity and the medium velocity. Figure 2a shows the wavefront calculated using the Split-Step Fourier (SSF) (phase-screen) method. Figure 2b presents the result using the hybrid pseudo-screen propagator, and Figure 1c is that by a traditional 65-degree finite-difference method implemented in the frequency-space domain. We observe that the SSF wavefront is only accurate for small-angle waves. While the *F-X* finite difference responded better for large-angle waves; the dispersion and artifacts are quite conspicuous. In contrast, the dual-domain Hybrid Pseudo-Screen Propagator performs quite well for large-angle waves.

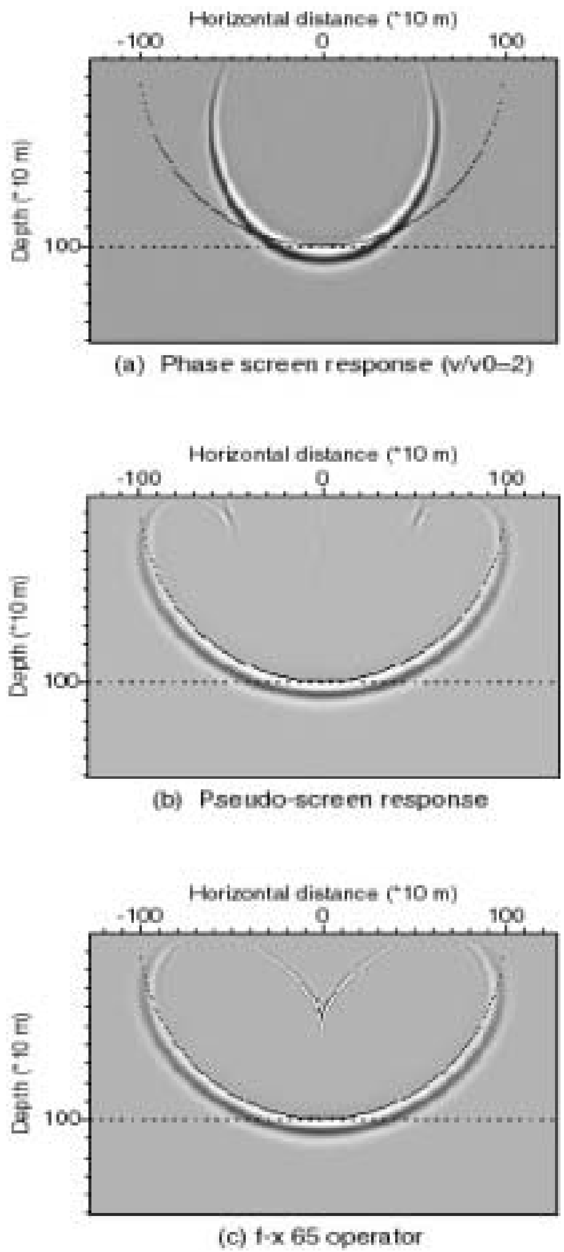

Figure 2
Impulse responses of three different migration operators: (a) phase-screen propagator; (b) hybrid pseudo-screen propagator; (c) 65 degree finite-difference propagator.

The amplitude-preserving feature of the new dual-domain propagators is demonstrated in Figure 3. Events with a dip angle up to 60° can be migrated with little distortion of amplitude. Even for the structures dipping up to 70°, only the deepest portion of the migrated image has significant distortion. In the lower panel of Figure 3, it is also shown that the amplitude is preserved even in the case of strong lateral velocity variations (100% perturbations).

The efficiency of the dual-domain wave-theory based methods is no longer a weak point in the 3-D case compared with the ray-theory based methods, such as Kirchhoff migration. For 2-D imaging, the dual-domain methods are generally a few times slower than the Kirchhoff method. However, for 3-D imaging, the situation is different: While the time for ray-tracing required in Kirchhoff migration increase as N^4 (N is the number of points in one dimension), the time for dual-domain methods increases as only $2N^2 \log_2 N$. When N is large in the 3-D case, the dual-domain methods are not necessarily slower than the ray methods. Further, the dual-domain methods can be formulated in the midpoint-offset coordinate system without difficulty (JIN and WU, 1999b), while a finite difference solution for this system has not been found. For marine data, the number of offsets is considerably smaller than the number of sources; thus there is a gain in efficiency when using an offset-domain migration formulation with dual-domain propagators.

4.4 Examples of GSP Migrations Applied to Different Data Sets

We present migration examples for 2-D and 3-D models to demonstrate the features and excellent performance of this approach. The first example is the 2-D prestack depth migration for the A-A′ profile of the SEG-EAEG salt model. The profile crosses many of the difficult structural elements in the model including steep, irregular shallow salt flanks, abrupt dip changes where the faults are located, strong velocity contrasts between the salt body and the surrounding medium (3–4 time differences). These pose an immense challenge to the conventional imaging methods. Figure 4 shows the imaging results of prestack depth migration using our Padé-screen propagator method (XIE and WU, 2000). We see that not only the salt body but also the subsalt structures were imaged clearly. The lateral and vertical resolutions are excellent.

The next example is the 2-D prestack migration image for the Marmousi model using the offset-domain pseudo-screen propagator (for theory see JIN and WU, 1999). The model contains very complicated geological features, especially shallow steep faults and an underlying high velocity lateral salt body intrusion. In addition, the model contains salt structure related traps and a reservoir structure beneath this complex geology. Figure 5c shows the result of ProMAX prestack Kirchhoff depth migration using finite-difference eikonal traveltimes. As expected, the multiple arrivals generated by this model cause the mislocation of reflections in complicated regions. Figure 5a is the image by 70° explicit finite-difference migration. Figure 5b

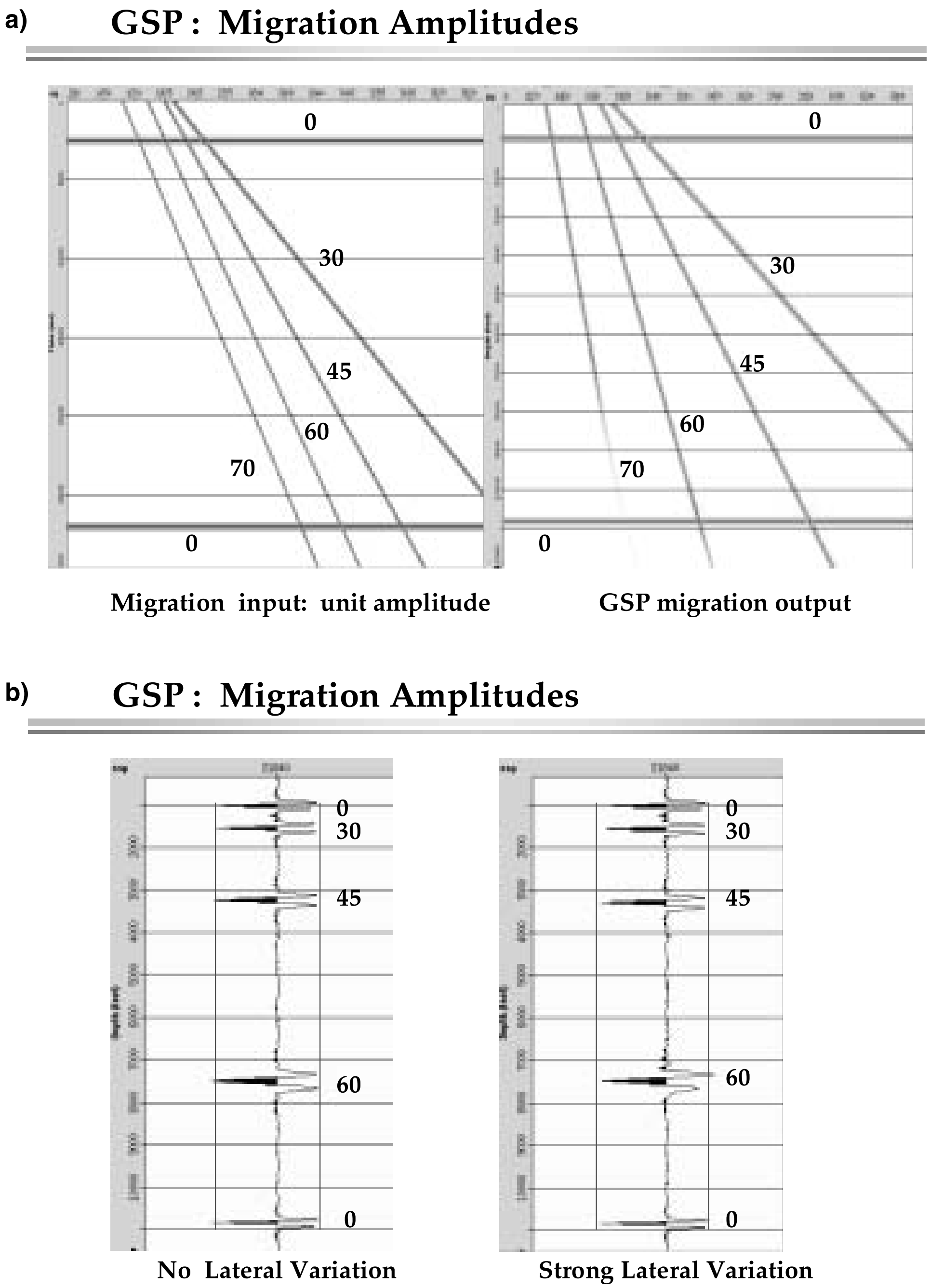

Figure 3
Amplitude preserving property of the wide-angle dual-domain migration methods. Top: Migration input and output for events with different dip-angles; Bottom: Traces showing the detailed amplitude and waveform information for the cases of no lateral variation and with strong lateral variation.

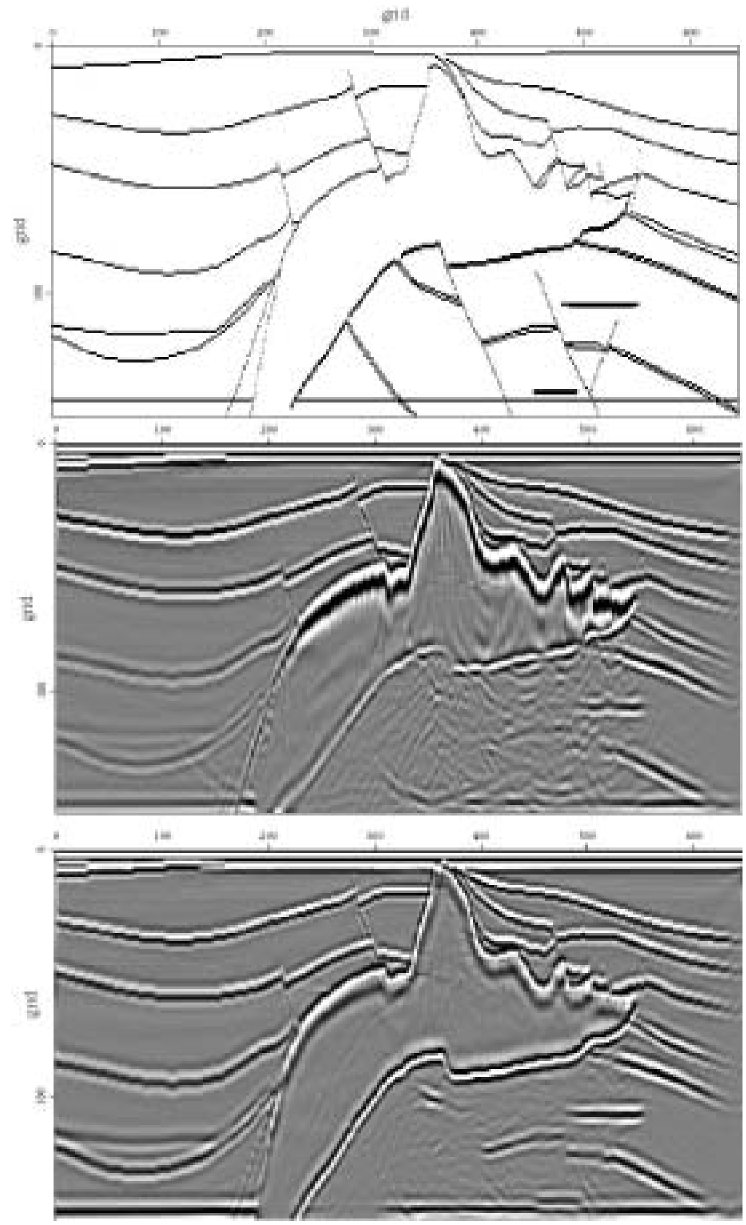

Figure 4
Prestack depth migration for A-A′ profile of the SEG-EAGE salt model using the wide-angle Padé-screen: Top: Reflection model; Middle: Image by phase-screen; Bottom: Image by Padé-screen.

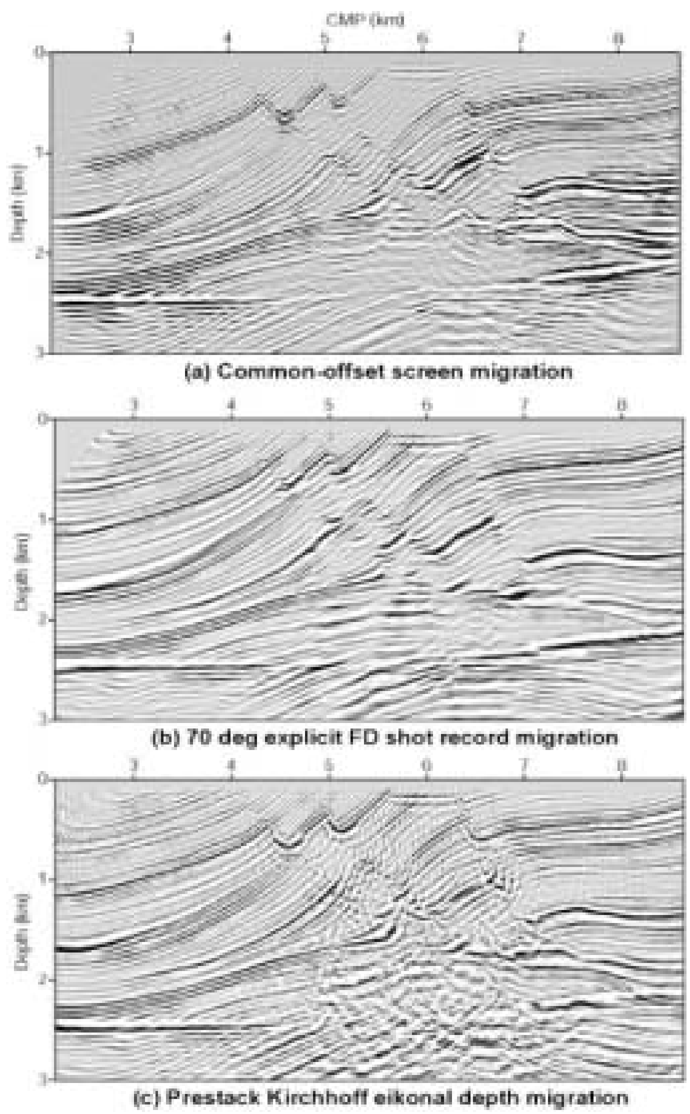

Figure 5
Prestack depth migration of Marmousi data set: (a) Offset-domain pseudo-screen migration. Forty-eight offset sections are used. (b) Image by 70 degree explicit finite-difference shot record migration. (c) Image by Kirchhoff migration.

shows the result of offset-domain pseudo-screen depth migration. Forty-eight offset sections with full offset range between 200 m and 2550 m were used in the test. The superiority of the image quality is apparent and it can delineate faults and reservoirs well enough to identify the major features.

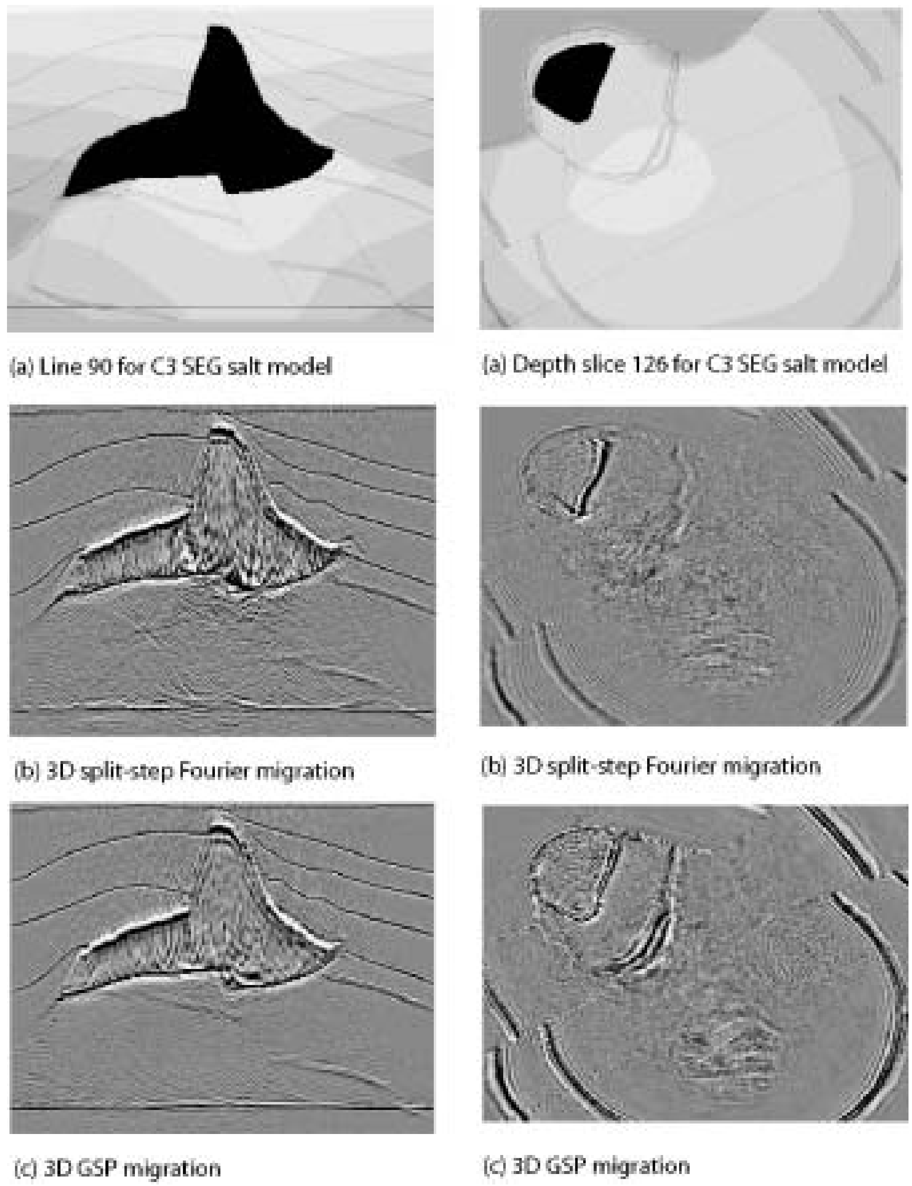

Figure 6
Comparison between images from different methods for the C3 subset of 3-D SEG Salt model. On the left is a vertical slice at line 90 and on the right is a horizontal slice at depth 126. Panels A, B and C are velocity models, images from the split-step Fourier method and images from the GSP migration, respectively.

Figure 6 provides an example of poststack 3-D migration for the C3 subset of the SEG-EAEG salt model. The synthetic data were generated by ARCO for a decimated model of 250×250 with a 40 m spacing using a finite-difference exploding reflector algorithm. Figure 6a presents a vertical profile (on the left) and a horizontal slice at depth grid 126 (on the right). In Figures 6b are shown the vertical cut (on the left) and horizontal slice (on the right) of the 3-D images reconstructed by split-step Fourier migration. Figures 6c are the reconstructed images by the hybrid pseudo-screen migration. We can see clearly the improved image quality of the GSP migration over the split-step Fourier migration, especially the improvements of resolution, image sharpness, fault delineation and noise reduction for subsalt structures.

Figure 7 shows a horizontal slice of the 3-D SEG salt model (top panels) and the images obtained from prestack migration of a subset of a portion of the numerical dataset, applying the wide-angle Padé-screen method (bottom panel) compared with the split-step Fourier method (middle panel). The data set is a common-source gather with a total of 45 shots. It can be seen clearly that the new wide-angle dual-domain method performs substantially better in imaging the faults and defining the sharp structural boundaries.

5. Modeling and Simulation Using Dual-domain Propagators

Because of the super wide-angle capacity of the new dual-domain propagators, they can be applied to modeling the wave propagation in complex media such as heterogeneous crustal waveguides (Wu *et al.*, 2000a,b), random media (Fehler and Huang, 2000), and other cases where forward scattering dominates. Combining the multi-forward scattering and single-backscattering approximations, the dual-domain propagators can be used to model the primary reflections in complex elastic media (Wu, 1994, 1996; Xie and Wu, 1995, 1996, 1998, 1999; Wild and Hudson, 1998; Wild *et al.*, 2000; Wu and Wu, 1998, 1999). Figure 8 illustrates an example of synthetic reflection seismograms using the thin-slab operator for the elastic French Model (top panel). The dark model has −20% perturbations relative to the surrounding medium for both *P*- and *S*-wave velocities. In the figure, the solid reference lines were calculated using a full-wave finite-difference algorithm. The dotted lines are the results of the thin-slab approximation, which is a dual-domain one-return propagator. Direct arrivals are not shown in the figure. It can be seen that the thin-slab operator can accurately calculate the backscattered waves extending to quite large angles.

6. Conclusions

Wide-angle dual-domain propagators, including the generalized screen propagators (GSP) have adaptive phase-space manipulations. In homogeneous regions the wavenumber-domain operation dominates; while in heterogeneous regions, it turns

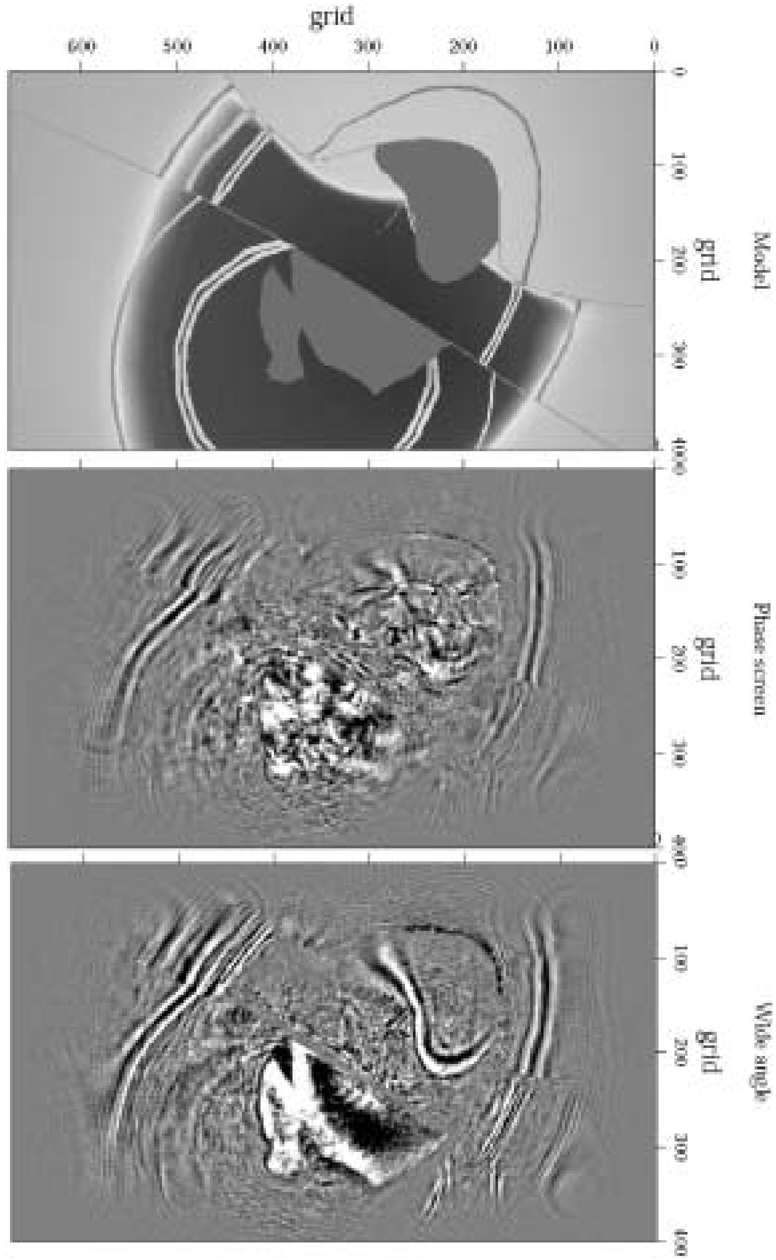

Figure 7
Comparison of 3-D prestack migration images using different methods. The horizontal slice is located at $Z = 2100$ m. From top to bottom are velocity model, migration image using the phase-screen method, and image using the wide-angle Padé-screen method, respectively.

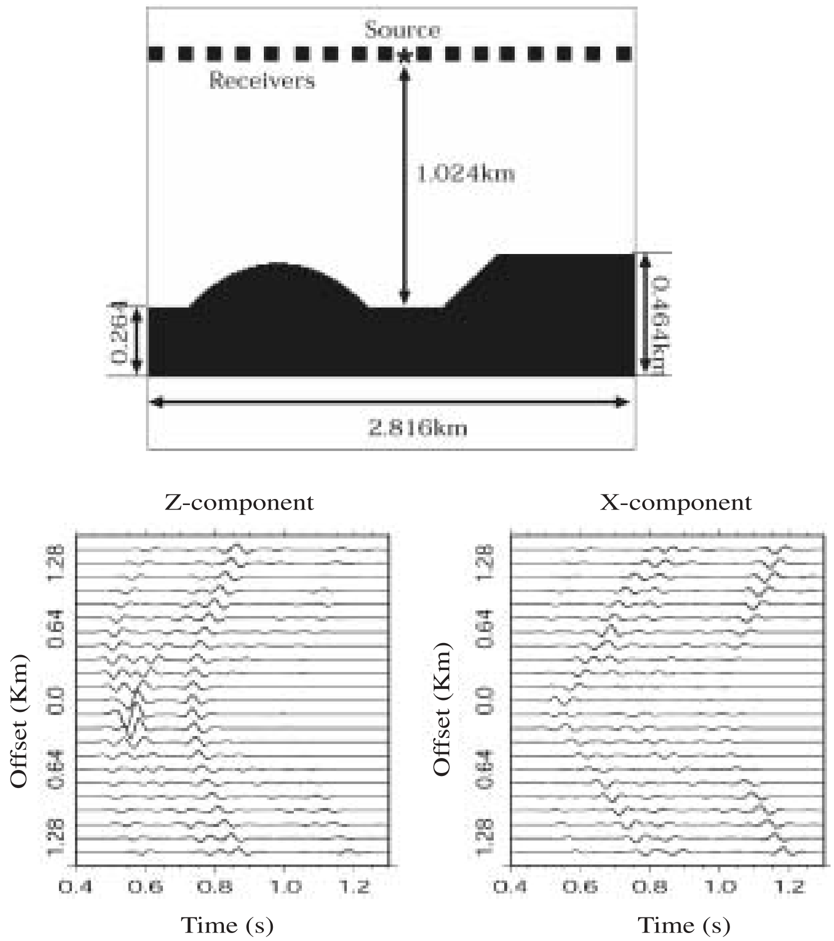

Figure 8
Synthetic reflection seismograms calculated by the thin-slab approximation (dotted lines) compared with those by finite-difference calculation (solid lines) for an elastic French model with an explosion source.

into weighted mixed domain operations. The weight is proportional to the strength of heterogeneity and space-domain operation may dominate. In summary, the wide-angle dual-domain propagators have the following features: **High-resolution** and **High-fidelity**: imaging based on wave theory; **High-speed**: one-way approximation + FFT implementation; **Super-wide-angle performance**: dual-domain adaptability to heterogeneities; **Reduced grid-dispersion**: hybrid FT-FD; **Midpoint-offset domain imaging capability**: high efficiency; **Dual-domain information available** at each step: convenient for imaging **velocity analysis** and **AVA** (amplitude versus angle) analysis.

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Appendix

Renormalization of Scattering Series and the De Wolf Approximation

The Lippmann-Schwinger equation

$$u = u^0 + k^2 \int_V d^3\vec{r}' g_0(\vec{r};\vec{r}')\varepsilon(\vec{r}')u(\vec{r}') \ , \tag{A.1}$$

where $\varepsilon(\vec{r}')$ is the equivalent body force for scattering or scattering potential as identified in the scattering theory, can be written symbolically as

$$u = u^0 + G_0\varepsilon u \tag{A.2}$$

where ε is a diagonal operator in space domain, and G_0 is a nondiagonal integral operator. If the reference medium is homogeneous, G_0 will be the volume integral with the Green's function $g_0(\vec{r};\vec{r}')$ as the kernel. A formal "solution" of (A.2) is

$$u = [1 - G_0\varepsilon]^{-1}u^0 \ . \tag{A.3}$$

If we expand (A.3) into a series by iteration, it will become an infinite scattering series which is the familiar Born series. The Born series may converge very slowly or become divergent for strong scattering. Now let us split the operations (the interaction between the medium and the wave) so that we can resort the scattering series and sum up certain sub-series theoretically to remove the divergent elements in the Born series. This is the intent of renormalization. Here we split the scattering potential operator into a forescattering and a backscattering operator: $\varepsilon = \varepsilon_f + \varepsilon_b$, such that $G_0\varepsilon_f$ and $G_0\varepsilon_b$ correspond to the forescattered and backscattered Born solutions, respectively. It can be seen from straightforward operator algebra that

$$u = [1 - G_f \varepsilon_b]^{-1} u_f \tag{A.4}$$

with

$$u_f = [1 - G_0 \varepsilon_f]^{-1} u_0$$
$$G_f = [1 - G_0 \varepsilon_f]^{-1} G_0$$

or in the form of

$$u = u_f + G_f \varepsilon_b u \tag{A.5}$$

with

$$u_f = u^0 + G_0 \varepsilon_f u_f$$
$$G_f = G_0 + G_0 \varepsilon_f G_f \quad .$$

In explicit form, (A.5) can be written as

$$u(\vec{r}) = u_f(\vec{r}) + \int_V d^3\vec{r}' g_f(\vec{r};\vec{r}') \varepsilon_b(\vec{r}') u(\vec{r}') \quad . \tag{A.6}$$

Equation (A.6) thus expresses $u(\vec{r})$ in terms of a renormalized forward propagated field u_f, and a scattered component due to a medium with scattering potential ε_b, and an effective forward propagator $g_f(\vec{r};\vec{r}')$ instead of the background Green's function g_0. Equation (A.6) can be solved with an iterative procedure. The first iteration will be the De Wolf approximation (33).

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